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Estimated Numerical Results and Simulation of the Plant Disease Model Incorporating Wind Strength and Insect Vector at Equilibrium

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Authors' contributions

This work was carried out in collaboration between all authors. Author ALMM designed the study, performed the analysis, wrote the protocol, and wrote the first draft of the manuscript. Author TO managed the analyses of the study. Author BO managed the literature searches. All authors read and approved the final manuscript.

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Abstract

Numerical simulations facilitate in the study of the behaviour of systems whose mathematical models are too complex to obtain analytical solutions. In this paper, we used assumed values of the model parameters and variable to simulate a non-linear deterministic model for plant vector borne diseases developed by Murwayi in [1]. Available models incorperate one climate change parameters at a time but our model includes the three temperature, precipitate and wind at ago. The non linear deterministic model included the climate change parameters with temperature and precipitation that influence the biting rate as (a) while wind as an agent of movement of the insect vector, emmigration and immigration is incorperatedin the model as $(\pm\theta)$. We obtained the basic reproductive number R_0 and carried out the normalized sensitivity analysis to obtain the positive and negative numerical sensitivity indices to determine the parameters that have the greatest and lowest impact on the basic reproductive number R_0 of the model. We used Matlab ODE 45 in built solver to simulate the dynamics of the model.

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Keywords: Basic reproduction number; numerical sensitivity analysis and simulation point.

1 Introduction

Various infectious plant diseases are spread by intermediary carriers (vector) hence are known as vector borne plant diseases [2]. Insects are the most common vectors with aphids causing 70% of the vector borne plant diseases [3].Some insect's vector like aphids are most specific and hence cause diseases in specific plants while others have a broad range of hosts hence cause/spread diseases in many/several different plant species[4] Survival and reproduction rates of the vector, time of the year and vector activities like biting rate, development rate and reproduction of the pathogen on the vector/host plant are some of the important transmission properties of plant vector borne diseases [5] insect vectors are therefore susceptible to climate change parameters like wind, precipitation and temperatures and these affect the transmission of vector borne plant diseases [2].

Host movement, immune response, the spatial age structure and population densities of plants are different from those of animals. Host movement, immune response and recovery are therefore usually not considered in plant epidemiology [4].

Infectious vector borne plant diseases destroy plants and animals. They affect livestock crops and humans and are therefore a threat to food security [6] and hence their control and eradication is of great economic and public health concern. Environmental effects and climate changes play an important role in the transmission of the insect vector borne diseases and hence should be taken into account [7].

The dynamics of diseases can significantly be understood in terms of the cause, progression under certain conditions and guidance in the choice of the intervention measure through mathematical models. The Mathematical modeling of plant diseases has its roots in the classical model for micro parasites host interaction in mammals formulated in 1927 by Kermack and Mckendrick [8]which was an improvement on Ross malaria model of 1916 [9].It began with Van Der Plank developed the first model of temporal development of epidemic plant diseases[10] followed by others like Fitzbonn [11], Holt [12], Jeger [13],Otim [14] Gourley [15],Elward [16] , Wei [17], Cui [18]. Some highlighted the importance of the direct interactions between the vectors dynamics with a focus on the dynamics of the vector host disease system without considering the dynamics of the elements of the vector while others focused on specific crops and their biological control.

Moore [19] developed a model to determine how a predator of the vector affects the prevalence of a vector-borne disease in the absence of predators, Wallace [20] modeled the spatiotemporal dynamics of African Cassava Mosaic disease in which he incorporated wind but as an agent to the movement of the vector while Lashari and Zama in their study of the global and back bifurcation developed a vector borne disease model with horizontal transmission in host population [21]. Zhou [7] improved on Moore's [19] model by developing a model to study the disease control threshold and limit cycles with persistence of disease or without disease. However, both Moore [19] and Zhou [7] focus was on total host population dividing host and vector into susceptible and infectious. Clearly the climate change variables have been modeled in isolation in any given model.

Recently, the research by Murwayi [1] developed a nonlinear deterministic mathematical model incorporating the climatic change variables to be used to evaluate the dynamics of vector- borne diseases in plants. The model is different from others in that it is nonlinear and combines the climate change variables. Temperature and precipitation which influence the biting rate (a) are considered under the parameter for biting rate (a) while wind was factored in as an agent to the movement of the vector, emigration or immigration hence incorporated in the model as $(\pm\theta)$. Since insects have very short life spans, the study developed a deterministic general model for the perennial plant disease with assumption that vectors reproduce very fast and attain equilibrium while the plant population is constant. After describing and developing the model, it was analyzed and the basic reproduction number (R_0) was obtained as 1.7 using the next generation method. The analytical global stability was determined using the Lyapnov method.

The analysis of nonlinear systems has to be approached using both analytic and computational schemes. Analytical methods provide existence results and some information on the qualitative behavior of the solutions like stability properties, asymptotic behavior and bifurcations. Computational schemes integrate the information by visualizing and completing the results of the analytical methods [22]. However neither the analytic nor computational analysis is sufficient for complete analysis, their synergic use is usually recommended to give an almost complete picture of the solution patterns.

Simulation is used based on the computational schemes to give a complete picture of the solution patterns and expand on what is given or not given by the quantitative analysis. Simulations include parameter sensitivity analysis, spatial modifications, analysis of the impact of source terms and initial condition variances [22]. Mathematical and computer simulations of models are the fundamental experimental tools of epidemiology. Repeatable experiments and accurate data are usually not available in epidemiology therefore mathematical models and computer simulations are used to do the necessary theoretical experiments with varying parameter values and data sets. Computer simulations quickly and easily depict what happens when one or several parameters are changed. They are also used to identify important combinations of parameters and essential aspects or variables in the model [22]

In this paper, we will simulate the plant disease dispersion model incorporating insect vector at equilibrium developed by Murwayi [1]. The organization of this paper is as follows: In section 2, we presented the model and a summary of the assumed values of the variables and parameters and used the values to evaluate the basic reproduction number. In section 3 we carry out the sensitivity analysis of the basic reproduction number R_0 . 1n section 4 we simulate the full model and briefly discuss then conclude in section 5.

2 Estimated Numerical Results and Simulations

The model has the following system of differential equations with the variables and parameters as defined in the Appendix 1

$$\frac{dS_P}{dt} = \mu_p N_P + \omega E_P - (k\lambda_P + \mu_p)S_P \quad (1),$$

$$\frac{dE_P}{dt} = k\lambda_P S_P - (\omega + \tau + \mu_p)E_P \quad (2),$$

$$\frac{dI_P}{dt} = \tau E_P - \mu_p I_P \quad (3),$$

$$\frac{dS_V}{dt} = \pi N_V \pm \theta S_V - (k\lambda_V + \delta + \mu_V)S_V \quad (4),$$

$$\frac{dI_V}{dt} = k\lambda_V S_V \pm \theta I_V - (\delta + \mu_V)I_V \quad (5),$$

Where $N_V(t) = S_V(t) + I_V(t), N_P(t) = S_P(t) + E_P(t) + I_P(t), \lambda_P = a\beta_1 I_V$ and $\lambda_V = a\beta_2(E_P + \eta I_P)$. Equations $(4-5)$ were considered at steady state which reduced the system of five equations

$(1-5)$ to the three equations (6-8) given below;

$$\frac{dS_P}{dt} = \mu_P N_P + \omega E_P - \left(\frac{a\beta_1 K^2 \pi N_V^* \lambda_V}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} + \mu_P\right) S_P \quad (6),$$

$$\frac{dE_P}{dt} = \frac{a\beta_1 K^2 \pi N_V^* \lambda_V S_P}{(k\lambda_V + \delta + \mu_V \pm \theta)(\delta + \mu_V \pm \theta)} - (\omega + \tau + \mu_P)E_P \quad (7),$$

$$\frac{dI_P}{dt} = \tau E_P - \mu_P I_P \quad (8),$$

The initial conditions were $S_P(0) = (S_P)_0$, $E_P(0) = (E_P)_0$ and $I_P(0) = (I_P)_0$. The expression of basic reproduction number (R_0) was obtained by the next generation method as $R_0 = \frac{a^2\beta_1\beta_2K^2\pi N_V^* N_P}{(\delta+\mu_V+\theta)^2(\omega+\tau+\mu_P)}\left\{1+\frac{\eta\tau}{\mu_P}\right\}$.

It was not possible to obtain the real secondary data in the literature therefore the following are the theoretical assumed data for the variables and parameters that was used for the purpose of validating the model.

Table 1. Summary of data and parameters values

Variables	Expression	Value	Source
$N_V^*(0)$		10000000	Assumed
$S_V^*(0)$		9962486	Assumed
$I_V^*(0)$	$I_V^* = \frac{k\lambda_V(0)\pi N_V^*}{(k\lambda_V(0)+\delta+\mu_V+\theta)(\delta+\mu_V+\theta)}$	≅37514	Assumed
$N_P(0)$		11	Assumed
$S_P(0)$,		10	Assumed
$E_P(0)$		1	Assumed
$I_P(0)$.		0	Assumed
Parameters			Assumed
π		0.0025	Assumed
ω		0.00078	Assumed
δ		0.00023	Assumed
μ_P		0.0034	Assumed
τ		0.0078	Assumed
μ_V		0.0056	Assumed
θ		0.056	Assumed
β_1		0.945	Assumed
β_2		0.832	Assumed
a		0.0076	Assumed
K		1	Assumed
η		0.6	Assumed
$\lambda_V(0)$	$\lambda_V(0) = a\beta_2(E_P(0)+\eta I_P(0))$	0.0063232	Assumed

$$\text{Secondary infections} = \frac{a^2\beta_1\beta_2K^2\pi N_V^* N_P}{(\delta+\mu_V+\theta)^2(\omega+\tau+\mu_P)}\left\{1+\frac{\eta\tau}{\mu_P}\right\} = 64803$$

The basic reproduction number R_0 estimates secondary infections when infected individuals are introduced in a completely susceptible population. Beginning with one exposed plant and 37,514 infected insects at equilibrium hence a total of 37515 infected individuals, the reproduction indicates they would produce 64803 new secondary infections. On average one infected individual produces about 1.7 secondary infected individuals.

$$E^0 = (S_P^0, E_P^0, I_P^0) = (N_P, 0, 0) = (11,0,0)$$

$$\lambda_V^* = \frac{\mu_P(\delta + \mu_V \pm \theta)^2(\omega + \tau + \mu_P)(R_0 - 1)}{(\omega + \tau + \mu_P)\{a\beta_1 K^2 \pi N_V^* + \mu_P[k(\delta + \mu_V \pm \theta)]\} - a\beta_1 K^2 \pi N_V^* \omega} = 0.0502$$

The study by Nakul [23] defined normalized forward sensitivity index of a variable Y that depends on the differentiability on a parameter Φ as

$$R_\Phi^Y = \frac{dY}{d\Phi} \times \frac{\Phi}{Y}$$

The assumed parameters in the Table 1 were used to evaluate the normalized numerical analysis for our model as summarized in Tables 2, 3 and 4. Positive numerical sensitivity index means that the parameters evaluated were directly related to the basic reproduction number.

Table 2. Summary of parameters with positive normalized indices

Parameter	**a**	**k**	N_P	N_V	β_1	β_2	η
Expression	2	2	1	1	1	1	$\frac{\eta\tau}{\eta\tau + \mu_P}$
Value	2	2	1	1	1	1	0.5792

The higher the positive value the more the impact the parameter has on the basic reproduction number. From the above table, the parameter a and k have the highest impact while η has the lowest impact.

The parameters in the table below had negative numerical sensitivity index which means they are directly related to the basic reproduction number.

Table 3. Parameter with negative values of normalized sensitive index

Parameter	τ	μ_P	ω
Expression	$\frac{\eta\tau}{\eta\tau + \mu_P} - \frac{\tau}{\tau + \omega + \mu_P}$	$-\frac{\eta\tau}{\eta\tau + \mu_P} - \frac{\mu_P}{\tau + \omega + \mu_P}$	$-\frac{\omega}{\tau + \omega + \mu_P}$
Value	-0.0719	-0.8630	-0.06511

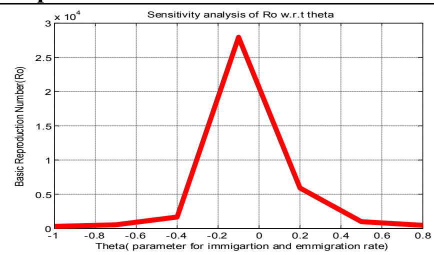
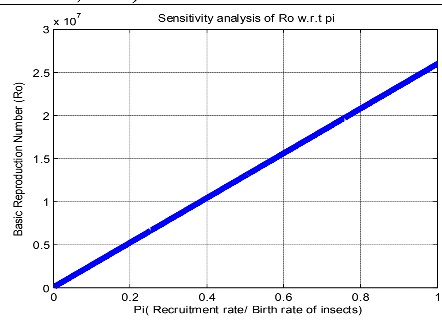
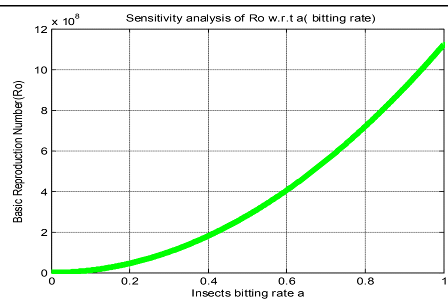
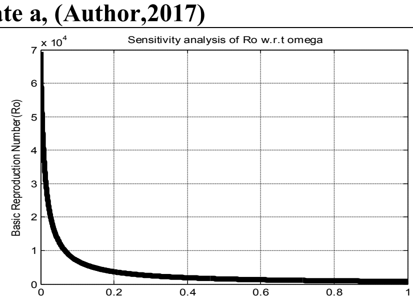
Table 4. Parameters values with negative normalized sensitive indices (continuation)

Parameter	θ	δ	μ_V	θ
Expression	$-\frac{2\theta}{\delta + \theta + \mu_V}$	$-\frac{2\delta}{\delta + \theta + \mu_V}$	$-\frac{2\mu_V}{\delta + \theta + \mu_V}$	$\frac{2\theta}{\delta - \theta + \mu_V}$
Value	-1.81142	-0.0074398	-0.1811418	-2.23241

The higher the negative value the more the impact the parameter has on the basic reproduction number. From the above table, the parameter θ has the highest impact while δ has the lowest impact.

3 Sensitivity Analysis of the Basic Reproduction Number Function

The basic reproduction number was plotted against changes in various parameters and the results obtained graphically as indicated below

Graph	**Description**
 **Fig. 1. Sensitivity analysis of Ro with respect to theta, (Author,2017)**	This is a graph of basic reproduction number (R_0) against the rate of emigration or immigration parameter θ. The result indicates R_0 is lowest when θ was below -0.4 and about 0.6. R_0 is highest when θ is about -0.1 More emigrations less contact and infections hence low R_0.More immigrations More vectors but host carrying capacity limiting infection low infection low R_0
 **Fig. 2. Sensitivity analysis of Ro with respect to pi, (Author,2017)**	This is a graph of basic reproduction number (R_0) against the rate of birth of insect parameter π. The result indicates R_0 is lowest when π is zero. R_0 increases with increase in π. Increase in insect recruitment means increased contact and transmission hence increased R_0
 **Fig. 3. Sensitivity analysis of Ro with respect to biting rate a, (Author,2017)**	This is a graph of basic reproduction number (R_0) against the biting rate parameter a. The result indicates R_0 is lowest when a is zero. R_0 increases with increase in a. Zero a means no biting and no disease transmission hence low R_0. As a increases so does the rates contact, biting and transmission hence R_0 increases.
 **Fig. 4. Sensitivity analysis of Ro with respect to omega, (Author,2017)**	This is a graph of basic reproduction number (R_0) against the rate of recovery from natural immunity parameter ω. The result indicates R_0 is lowest when ω is zero. R_0 decreases with increase in ω.Increased immunity leads to less or no infection with time so low R_0.

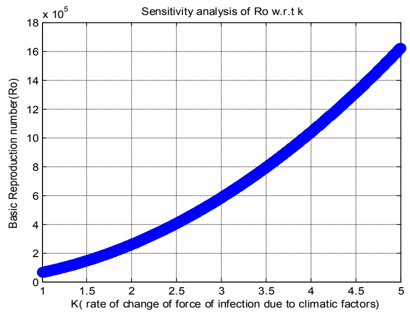


Fig. 5. Sensitivity analysis of Ro with respect effect of environmental factor k to force of infection, (Author,2017)

This is a graph of basic reproduction number (R_0) against the rate of effect of environment on force of infection parameter k. The result indicates R_0 is lowest when k is zero. R_0 increases with increase in k . Increased favourable environmental conditions on force of infection k means more contact and biting more infections hence increasing R_0.

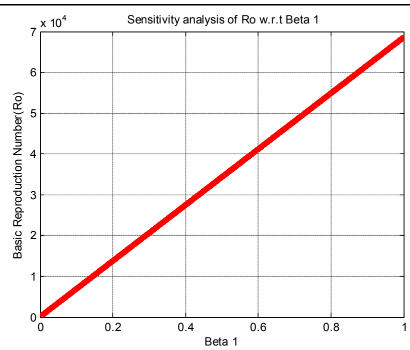


Fig. 6. Sensitivity analysis of Ro with respect to Beta 1, (Author,2017)

This is a graph of basic reproduction number (R_0) against the infection rates for a plants β_1 .The result indicates R_0 is lowest when β_1 is zero. R_0 increases with increase in β_1.

Low/ zero rate of infection fewer infected individuals hence lower R_0.

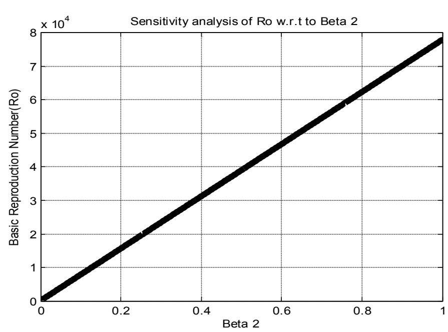
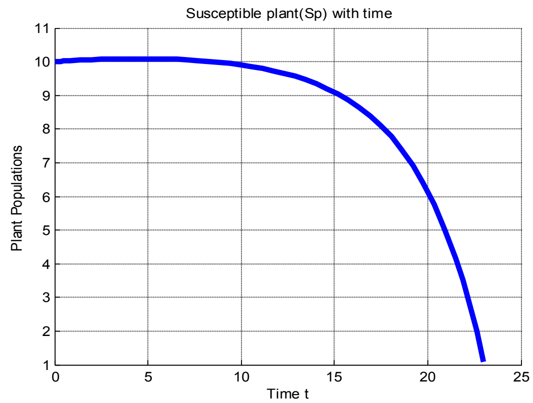
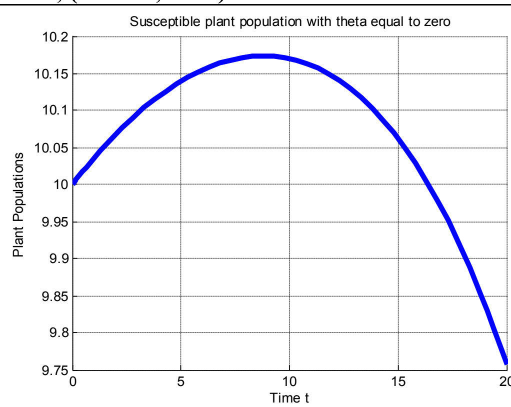
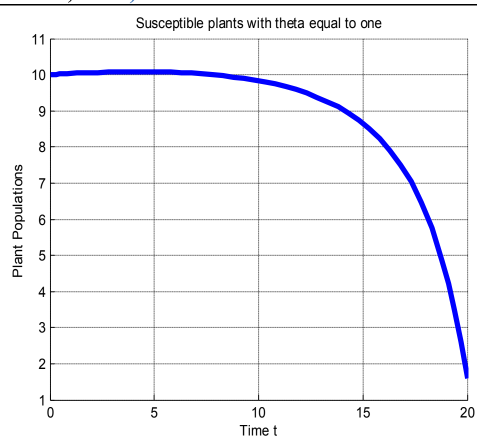


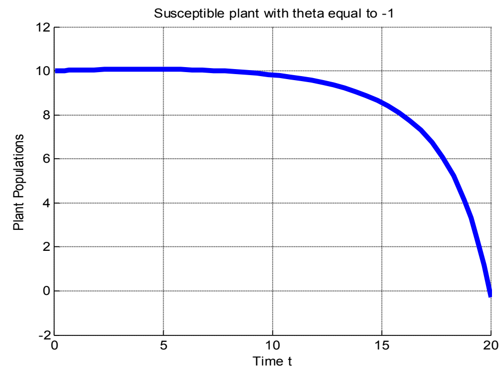
Fig. 7. Sensitivity analysis of Ro with respect to Beta 2, (Author,2017)

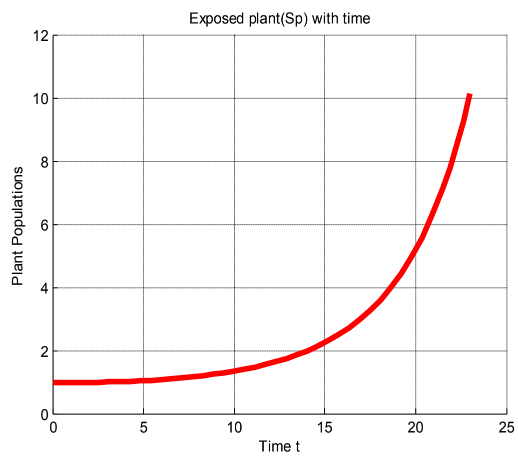
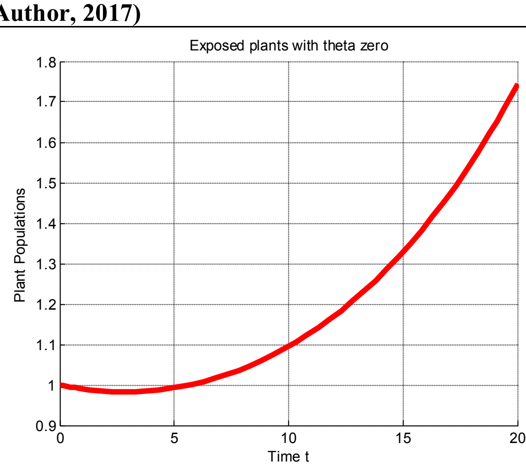
This is a graph of basic reproduction number (R_0) against the infection rates for vectors β_2.The result indicates R_0 is lowest when β_2 is zero. R_0 increases with increase in β_2. Infective vectors pick the pathogens from infected plants hence when their infection rate is low the transmission of the disease is also low and so is R_0 and vice versa

4 Simulation of the Model

The dynamics of the numerical results of the model [(6)-(8)] was simulated using Matlab ODE 45 in built solver to obtain the following graphs

0.056	 **Fig. 8. Simulation of susceptible plants with theta= 0.056, (Author, 2017)**	The parameters and data in subsection 4.3.1 were used to simulate the susceptible population with θ = 0.056. The graph indicates that susceptible plant would reduce from 10 to about 1. A positive θ means immigration of the insect vectors , more infection hence reduced susceptible host plants
0	 **Fig. 9. Simulation of susceptible plants with theta=0, (Author, 2017)**	The parameters and data in subsection 4.3.1 was used to simulate the susceptible population with θ = 0. The graph indicates that susceptible plant would reduce from 10 to about 0. There are no immigrations or emigrations but the vectors are present, infections occurring hence susceptible plants transiting to exposed
1	 **Fig. 10. Simulation of susceptible plants with theta=1, (Author, 2017)**	The parameters and data in subsection 4.3.1 was used to simulate the susceptible population with θ = 1. The graph indicates that susceptible plant would reduce from 10 to about 1. Increased immigration means more infestation and infection hence reduced susceptibles

-1	 **Fig. 11. Simulation of susceptible plants with theta=-1, (Author, 2017)**	The parameters and data in subsection 4.3.1 was used to simulate the susceptible population with θ = -1. The graph indicates that susceptible plant would reduce from 10 to about 0.Higher negative θ means more immigrations but contact and biting has already occurred so susceptible host plants already transiting to exposed

θ value	Exposed population	Description
0.056	 **Fig. 12. Simulation of exposed plants with theta=0.056, (Author, 2017)**	The parameters and data in subsection 4.3.1 was used to simulate the Exposed population with θ = 0.056. The graph indicates that exposed plant would rise from 1 to about 8. Increased immigration means increased contact , biting and disease transmission so susceptible plants become exposed hence the rise.
0	 **Fig. 13. Simulation of the exposed plants with theta equal to zero, (Author,2017)**	The parameters and data in subsection 4.3.1 was used to simulate the Exposed population with θ = 0. The graph indicates that exposed plant would rise from 1 to close to about 2. No emigrations and no immigration but vectors present contact with susceptible leading to transition to exposed hence the rise in exposed

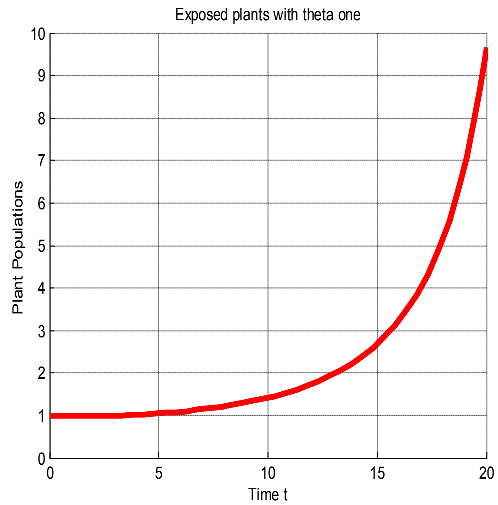
<table>
<tr><td>1</td><td>

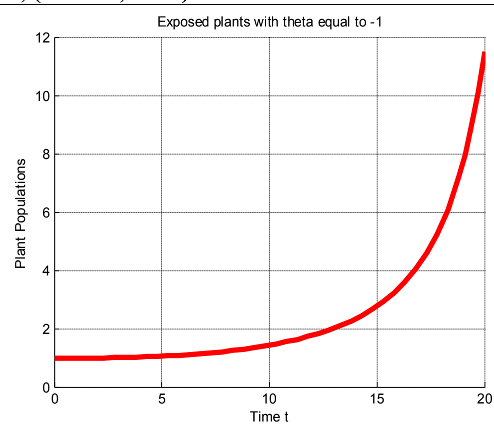
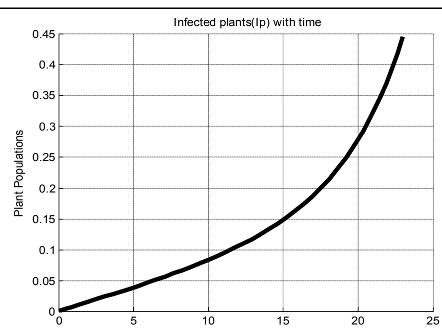
Fig. 14. Simulation of exposed plants with theta equal to one, (Author, 2017)</td><td>The parameters and data in subsection 4.3.1 was used to simulate the Exposed population with θ = 1. The graph indicates that exposed plant would rise from 1 to close to about 10.Increased invasion increased exposure hence the exposed.</td></tr>
<tr><td>-1</td><td>

Fig. 15. Simulation of exposed plants with theta equal to negative one, (Author, 2017)</td><td>The parameters and data in subsection 4.3.1 was used to simulate the Exposed population with θ = -1. The graph indicates that exposed plant would rise from 1 to close to about 12.Increased emigration but after contact and biting of the host plants.</td></tr>
</table>

θ value	**Infected population**	**Description**
0.056	 **Fig. 16. Simulation of Infected plants with theta equal to 0.056, (Author, 2017)**	The parameters and data in subsection 4.3.1 was used to simulate the infected population with θ = 0.056. The graph indicates that infected plants would rise from 0 to about 1. Force of infection is positive hence despite the emigration more plants are getting infected

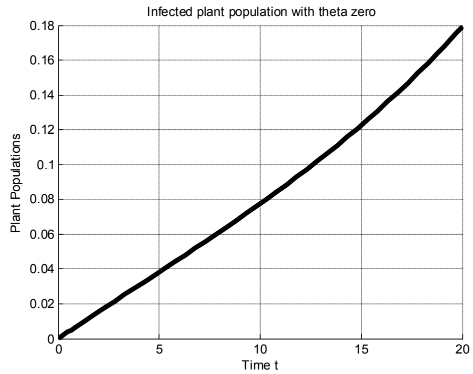
<table>
<tr>
<td>0</td>
<td>


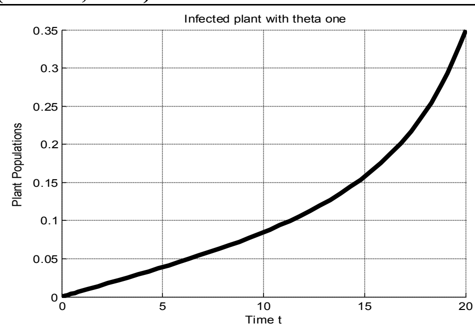
Fig. 17. Simulation of infected plants with theta equal zero, (Author, 2017)
</td>
<td>The parameters and data in subsection 4.3.1 was used to simulate the infected population with θ = 0. The graph indicates that infected plants would rise from 0 to about 0.18. Disease has already invaded with the presence of the vectors.</td>
</tr>
<tr>
<td>1</td>
<td>


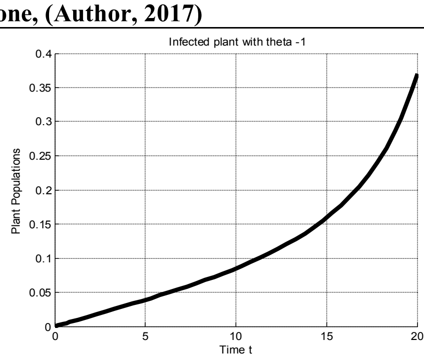
Fig. 18. Simulation of infected plants with theta equal to one, (Author, 2017)
</td>
<td>The parameters and data in subsection 4.3.1 was used to simulate the infected population with θ = 1. The graph indicates that infected plant would rise from 0 to 0.35. An influx of vectors means increased infections and infected plants.</td>
</tr>
<tr>
<td>-1</td>
<td>


Fig. 19. Simulation of infected plants with theta equal to negative one, (Author, 2017)
</td>
<td>The parameters and data in subsection 4.3.1 was used to simulate the infected population with θ = -1. The graph indicates that infected plant would rise from 0 to above 0.35. There are insect vector emigrations but plants infected hence exposed transiting to the infected.</td>
</tr>
</table>

Simulation of full model with variation of $\theta = \mathbf{1}\ \boldsymbol{and}\ \boldsymbol{K} = \mathbf{1.05}$

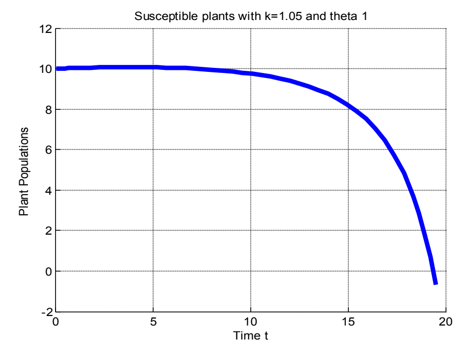


Fig. 20. Simulation of susceptible plants with theta equal to one and environmental coefficient equal to 1.05, (Author, 2017)

The parameters and data in subsection 4.3.1 was used to simulate the susceptible population with $\theta = 1$ and $K = 1.05$. The graph indicates that susceptible plant would reduce from 10 to 0. Vectors entry and favourable force of infection means more susceptible plants transit to exposed hence the decline in their population

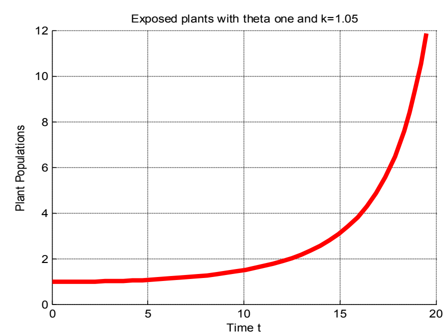


Fig. 21. Simulation of exposed plants with theta equal to one and k=1.05, (Author, 2017)

The parameters and data in subsection 4.3.1 was used to simulate the Exposed population with $\theta = 1$ and $K = 1.05$. The graph indicates that exposed plant would rise from 1 to close to about 9. Vectors entry and favourable force of infection means more susceptible plants transit to exposed hence the rise in their population

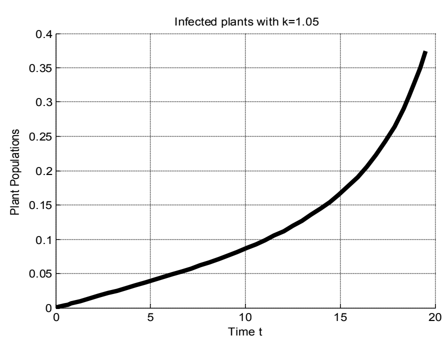


Fig. 22. Simulation of infected plants with θ equal to one and k=1.05, (Author, 2017)

The parameters and data in subsection 4.3.1 was used to simulate the infected population with $\theta = 1$ and $K = 1.05$. The graph indicates that infected plant would rise from 0 to above 0.35. Vectors entry and favourable force of infection means increased contact and more exposed plants transit to infected hence the rise in their population

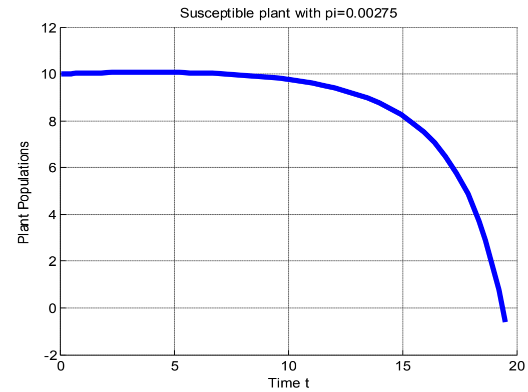


Fig. 23. Simulation of susceptible plants with pi=0.00275, theta equal to one and k=1, (Author, 2017)

The parameters and data in subsection 4.3.1 was used to simulate the susceptible population with $\pi = 0.00275, \theta = 1$ and $K = 1$

The graph indicates that susceptible plant would reduce from 10 to 0 with time. The three positive rates favour contact and transmission of the disease leading to the reduction in susceptible plants.

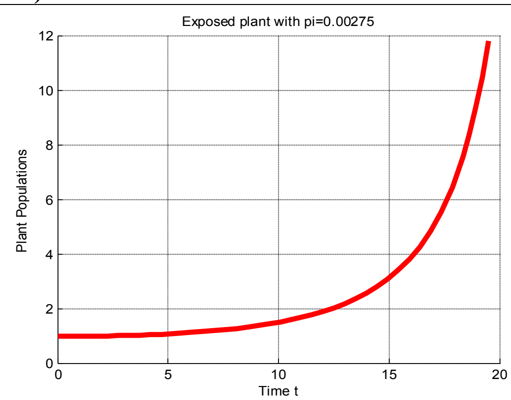


Fig. 24. Simulation of exposed plants with pi= 0.00275, theta equal to negative one and k=1, (Author, 2017)

The parameters and data in subsection 4.3.1 were used to simulate the Exposed population with θ = -1. The graph indicates that exposed plant would rise from 1 to close to about 12. The positive rates favour contact and transmission of the disease susceptible plants transit to exposed the rise despite the emigrations

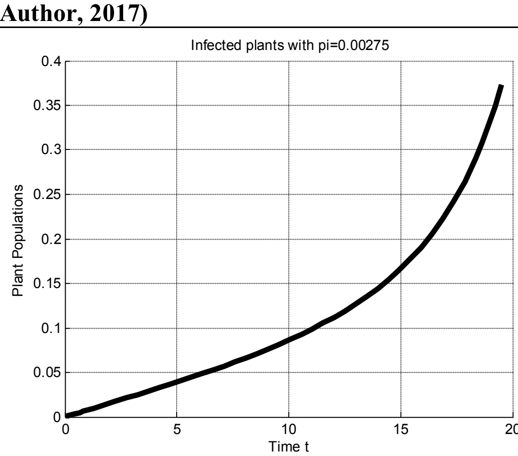


Fig. 25. Simulation of infected plants with pi=0.00275, theta equal to one and k=1, (Author, 2017)

The parameters and data in subsection 4.3.1 were used to simulate the infected population with $= 0.00275,$

$$\theta = 1 \text{ and } K = 1$$

The graph indicates that infected plants would rise from 0 to above 0.35. Despite the high emigration of the insect vector. The three positive rates favour contact and transmission of the disease leading to the exposed plants that transit to the infected .

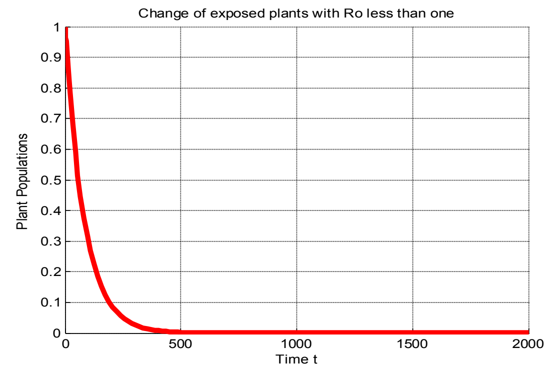


Fig. 26. Simulation of exposed plants with Ro less than one, (Author, 2017)

The simulation graph for the change of exposed plant with time indicates that the condition $R_0 < 1$ is a necessary and sufficient condition to reduce exposed plants to zero with time.

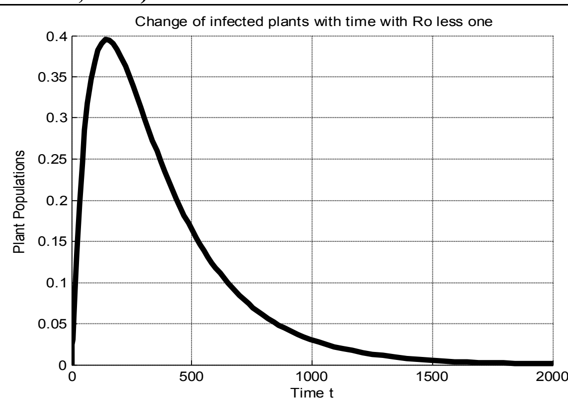


Fig. 27. Simulation of the infected plants with Ro less than one, (Author, 2017)

The simulation graph for the change of infected plant with time indicates that the condition $R_0 < 1$ is a necessary and sufficient to reduce infected plants to zero with time. The disease fails to invade

5 Conclusion

The estimated numerical value for R_0 was obtained as 1.7 which means that when one infectious plant with insect population at equilibrium is introduced in a field of susceptible plants, it is likely to infect 1.7 other plants. The normalized sensitivity analysis indicates that lowering the force of infection and biting rate hold the promise to reducing the spread insect vector borne plant diseases. Numerical simulations using Matlab in built ODE solver showed that whenever $R_0 < 1$ the infection would die out but infection would persist whenever $R_0 > 1$. Simulation also showed that changes in various parameters would affect the dynamics of the infected plants. Government policy makers and farmers have no control over climate changes; immigration and emigration of insects, therefore lowering the force of infection and biting rate, inducing insect deaths among others hold great promise in reducing the spread of the disease and lowering the burden of infection. This study developed a nonlinear deterministic model, future studies can consider stochastic model.

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Competing Interests

Authors have declared that no competing interests exist.

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Appendices

Appendix 1: Summary of variables and parameters

Variables	Descriptions
$N_V(t)$	The total population of the vectors
$N_P(t)$	The total population of the plants
$S_P(t)$,	The population of susceptible plant
$E_P(t)$	The population of exposed plant
$I_P(t)$.	The population of infected plant
$S_V(t)$	The population of susceptible vectors
$I_V(t)$	The population of infected vectors
Parameters	**Descriptions**
π	The recruitment rate of vectors
μ_P	The recruitment rate of plants
ω	The rates at which $E_P(t)$ recover naturally to $S_P(t)$.
δ	The rate at which climate induced death occur in both $S_V(t)$ and $I_V(t)$
μ_P	The constant natural death rate in subclasses $S_P(t), E_P(t)$ *and* $I_P(t)$
τ	The rate at which $E_P(t)$ progresses to $I_P(t)$.
μ_V	The constant natural death rate in subclasses $S_V(t)$ *and* $I_V(t)$
θ	The immigration and emigration rate of vector (this implies that the rate can be either be positive or negative).
β_1	The infection rates for plants
β_2	The infection rates for vectors
a	The vector biting rate
K	This is rate of change force of infection due to climatic factors. It is assumed $K \geq 1$
λ_P	The number of individual plants which become infected at given time(force of infection of plants)
λ_V	The number of individual plants which become infected at given time(force of infection of plants)

Appendix 2: Definition of terms

Term	Definition
Susceptible population	The populations who are free of infection but are at a risk of contracting the infection.
Exposed population	The populations that has contracted the infection but are at a lower risk of transmitting it and/or with no signs of infection
Infected population	The population with the disease causing pathogen and capable of transmitting the infection to other plants or vectors on contact
Basic reproduction number/ratio	The number of cases one generates on average over the course of infectious period in a completely susceptible population
Bifurcation	It occurs when a smooth change is made to parameter values causes the system to change its behavior
Disease free equilibrium point	It is infection free point
Endemic equilibrium point	The stationary point at which the disease has completely invaded the population

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