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The n^{th} Power Transformation of the Error Component of the Multiplicative Time Series Model

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Authors' contributions

This work was carried out in collaboration between all authors. Author AOD designed the study, wrote the protocol and supervised the work. Authors AOD and ELO carried out all laboratories work and performed the statistical analysis. Author ELO managed the analyses of the study. Author DCC wrote the first draft of the manuscript. Author ELO managed the literature searches and edited the manuscript. All authors read and approved the final manuscript.

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Abstract

In this paper the author(s) present derivations for the mean and variance of the n^{th} power transformation of the error component of the multiplicative time series model. As a general rule to any power transformation. Some of the published transformations like the square root and the inverse were used to validate the results obtained. The results showed that they conformed to the general rule.

Keywords: Power transformations; probability density; function; error component; multiplicative time series.

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1 Introduction

The pdf of the normal distribution is given in Uche [1] as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \; e^{-\frac{1}{2}\left(\frac{x-1}{\sigma}\right)^2}, x \geq 0, \sigma^2 > 0 \tag{1}$$

The error component e_t of the multiplicative time series model has a pdf $N(1, \sigma^2)$ where $e_t > 0$, Iwueze [2] established the distribution of the left-truncated normal distribution and is given by

$$f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left[1 - \Phi\left(\frac{-1}{\sigma}\right)\right]}, x \geq 0, \sigma^2 > 0 \tag{2}$$

With mean $E(X)$ and variance $Var(X)$ given by

$$E(X) = 1 + \frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}, x \geq 0, \sigma^2 > 0 \tag{3}$$

and

$$Var(X) = \frac{\sigma^2}{2\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(\left[1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right]\right) - \frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} - \left[\frac{\sigma e^{-\frac{1}{2}\sigma^2}}{\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}\right]^2 \tag{4}$$

respectively.

Iwueze [2] examined some implications of truncating the $N(1, \sigma^2)$ to the left. Which include:

(i) That the truncated values are always greater or equal to the non-truncated values for all values of σ. However, the two stochastic variables behave alike in the interval $\sigma<0.30$. It follows from the analysis that the 0.001 limits may be used to give practical assurance that the truncated (truncation at zero) values from the $N(1, \sigma^2)$ distribution are all positive. In the interval $\sigma<0.30$, the truncated and the non-truncated variables have the same mean equal to 1 and variance equal to σ^2.

(ii) The most important implication of truncating the $N(1, \sigma^2)$ distribution to the left at zero is in descriptive modelling of time series data, where the logarithmic transform of the truncated distribution is equally assumed to have mean zero and some finite variance. It was noted that the logarithmic transform will have mean zero and the same variance as both the original $N(1, \sigma^2)$ distribution and its truncated distribution in the interval $\sigma<0.10$.

The truncated normal distribution has gained much acceptance in various fields of human endeavours, these include inventory management, regression analysis, operation management, time series analysis and so on. Johnson and Thomopoules [3] considered the use of the left truncated distribution for improving achieved service levels. They presented the table of the cumulative distribution function of the left truncated normal distribution and derived the characteristic parameters of the distribution, and also presented the table of the partial expectation of the left truncated normal distribution.

A time series is a collection of ordered observation made sequentially in time. Examples abound in Sciences, Engineering, Economics, etc and methods of analysing time series constitute a vital area in the field of Statistics.

According to Spiegel and Stephens [4] the general time series model is always considered as a mixture of four major components, namely the Trend T_t, Seasonal variations S_t, Cyclical variations $C_{t,}$ and Irregular variations or Random Movements e_t. Hence classifications of the time series model are

Multiplicative model: $X_t = T_t S_t C_t e_t$ (5)

Additive model: $X_t = T_t + S_t + C_t + e_t$ (6)

Mixed model: $X_t = T_t S_t C_t + e_t$ (7)

In short term series the trend and cyclical components are merged to give the trend-cycle component; hence equation (5) through (7) can be rewritten as

$$X_t = M_t S_t e_t \quad (8)$$

$$X_t = M_t + S_t + e_t \quad (9)$$

$$X_t = M_t S_t + e_t \quad (10)$$

where M_t is the trend cycle component and e_t is independent identically distributed (iid).

normal errors with mean 1 and variance $\sigma^2 > 0$ $[e_t \sim N(1, \sigma^2)]$.

Most data in real life are non negative in nature, example, annual rainfall in a given city, sales, school enrolments, reported cases of crimes, accidents, the list is endless.

2 Data Transformation

Data transformation is a mathematical operation that changes or modifies the value and shape of a distribution function. Reasons for transformation include stabilizing variance, normalizing, reducing the effect of outliers, making a measurement scale more meaningful, and to linearize a relationship. For more references see Bartlett [5] Box and Cox [6], etc.

Many time series analysis assume normality and it is well known that variance stabilization improves normality of the series. The most popular and common transformation are the logarithm transformation and the power transformations (square, square root, inverse, inverse square, and inverse square root). It is important to note that, if we apply the n^{th} power transformation on model (8), we still obtain a multiplicative time series model given by

$$Y_t^n = M_t^n S_t^n e_t^n = M_t^* S_t^* e_t^* \quad (11)$$

where $M_t^n = M_t^*$, $S_t^n = S_t^*$, $e_t^n = e_t^*$

Several studies abound in statistical literature on effects of power transformations on the error component of a multiplicative time series model whose error component is classified under the characteristics given in equation (3). The sole aim of such studies is to establish the conditions for successful transformation. A successful transformation is achieved when the derivable statistical properties of a data set remain unchanged after transformation, there basic properties or assumptions of interest for the studies are (i) unit mean and (ii) constant variance. Also Nwosu et al. [7] studied the effects of inverse and square root transformation respectively on the error component of the same model and discovered that the inverse transform $Y = \frac{1}{e_t}$ can be assumed to be normally distributed with mean, one and the same variance provided $\sigma < 0.07$. Similarly Otuonye et al. [8] discovered that the square root transformation $\sqrt{e_t}$ can be assumed to be normally distributed with unit mean and variance $4\sigma^2$, for $\sigma_1 \leq 0.3$ where σ_1^2 is the variance of the original error component before transformation.

Ibeh et al. [9] studied the inverse square transformation of the error component of the multiplicative time series model, the results of the research showed that the basic assumptions of the error term of the multiplicative model which is normally distributed with mean 1 and finite variance can only be maintained if the standard deviation of the untransformed error term is less than or equal to 0.07 ($\sigma \leq 0.07$). The study also revealed that the variance of the transformed error term is 4 times the variance of the untransformed for $\sigma \leq 0.07$.

Ajibade et al. [10] studied the distribution of the inverse square root transformed error component of the multiplication time series model and found out that the means are the same and variance $Var(e_t^*) \approx \frac{1}{4} Var(e_t)$ for $\sigma \leq 0$.

In this paper a general rule expression for the pdf, mean and variance of n^{th} power distribution was used to verify some published power transformations, example, the square root and the inverse transformation.

2.1 Derivation of the probability density function (pdf) of n^{th} power transformation

Given

$$y = x^n \qquad \text{where } X = [e_t \sim N(1, \sigma^2)]$$

$$=> x = y^{\frac{1}{n}} \qquad => \frac{dx}{dy} = \frac{1}{n} y^{\frac{1}{n}-1}$$

$$\text{So } f(y) = f(x) \left|\frac{dx}{dy}\right|$$

$$f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]}, x \geq 0, \sigma^2 > 0 \qquad (12)$$

$$f(y) = \frac{e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} . \left|\frac{1}{n} . y^{\frac{1}{n}-1}\right|$$

$$f(y) = \frac{\frac{1}{n} . y^{\frac{1}{n}-1} \, e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} \qquad (13)$$

We now show that it is a proper pdf

$$i.e. \int_0^\infty f(y)dy = 1 \qquad (14)$$

$$let\ u = \frac{y^{\frac{1}{n}}-1}{\sigma}, \quad \frac{du}{dy} = \frac{1}{\sigma} . \frac{1}{n} y^{\frac{1}{n}-1}\ and\ dy = \frac{n\sigma}{y^{\frac{1}{n}-1}} du\ , \quad -\frac{1}{\sigma} < u < \infty$$

$$\int_{0}^{\infty} f(y)dy = \int_{-\frac{1}{\sigma}}^{\infty} \frac{\frac{1}{n}.y^{\frac{1}{n}-1}\, e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} \frac{n\sigma}{y^{\frac{1}{n}-1}} du \tag{15}$$

$$= \int_{-\frac{1}{\sigma}}^{\infty} \frac{e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2}}{\sqrt{2\pi}\left[1-\Phi\left(\frac{-1}{\sigma}\right)\right]} du$$

$$= \frac{1}{1-\Phi\left(-\frac{1}{\sigma}\right)} \int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du = \frac{1}{1-\Phi\left(-\frac{1}{\sigma}\right)} P_r\left(u > -\frac{1}{\sigma^2}\right)$$

But $\quad P_r\left(u > -\frac{1}{\sigma^2}\right) = 1-\Phi\left(-\frac{1}{\sigma}\right)$

$$\therefore \int_{0}^{\infty} f(y)dy = \frac{1-\Phi\left(-\frac{1}{\sigma}\right)}{1-\Phi\left(-\frac{1}{\sigma}\right)} = 1 \tag{16}$$

Therefore (15) is a proper pdf.

2.2 Derivation of the mean of the n^{th} power transformation

From definition $E(Y) = \int_0^{\infty} y f(y) dy$

$$=> E(Y) = \int_{0}^{\infty} \frac{\frac{1}{n}.y^{\frac{1}{n}-1}\, e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} dy$$

$$= \frac{1}{\sigma\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} \int_{0}^{\infty} \frac{1}{n} y^{\frac{1}{n}} e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2} dy \tag{17}$$

$$let\ u = \frac{y^{\frac{1}{n}}-1}{\sigma}, => \ \frac{du}{dy} = \frac{1}{\sigma}.\frac{1}{n} y^{\frac{1}{n}-1}\ and\ dy = \frac{n\sigma}{y^{\frac{1}{n}-1}} du\,, \quad -\frac{1}{\sigma} < u < \infty$$

Substitute (17) in (15), we have

$$E(Y) = \frac{1}{\sigma\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} \int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{n} y^{\frac{1}{n}} e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2} . \frac{n\sigma}{y^{\frac{1}{n}-1}} du$$

$$= \frac{1}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} \int_{-\frac{1}{\sigma}}^{\infty} y e^{-\frac{1}{2}u^2} du$$

$$= \frac{1}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} \int_{-\frac{1}{\sigma}}^{\infty} (1+\sigma u)^n e^{-\frac{1}{2}u^2} du \tag{18}$$

Recalled from binomial series

$$(1+\sigma u)^n = 1 + \binom{n}{1}\sigma u + \binom{n}{2}(\sigma u)^2 + \binom{n}{3}(\sigma u)^3 + \binom{n}{4}(\sigma u)^4 + \cdots$$

$$=> \int_{-\frac{1}{\sigma}}^{\infty} (1+\sigma u)^n e^{-\frac{1}{2}u^2} du$$

$$= \int_{-\frac{1}{\sigma}}^{\infty} e^{-\frac{u^2}{2}} du + \binom{n}{1}\sigma \int_{-\frac{1}{\sigma}}^{\infty} u e^{-\frac{u^2}{2}} du + \binom{n}{2}\sigma^2 \int_{-\frac{1}{\sigma}}^{\infty} u^2 e^{-\frac{u^2}{2}} du + \binom{n}{3}\sigma^3 \int_{-\frac{1}{\sigma}}^{\infty} u^3 e^{-\frac{u^2}{2}} du$$

$$+ \binom{n}{4}\sigma^4 \int_{-\frac{1}{\sigma}}^{\infty} u^4 e^{-\frac{u^2}{2}} du + \ldots + \binom{n}{n}\sigma^n \int_{-\frac{1}{\sigma}}^{\infty} u^n e^{-\frac{u^2}{2}} du$$

$$=> E(Y) = \frac{1}{\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} \left[\int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du + n\sigma \int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} u e^{-\frac{u^2}{2}} du + \frac{n(n-1)}{2!}\sigma^2 \int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} u^2 e^{-\frac{u^2}{2}} du \right.$$

$$+ \frac{n(n-1)(n-2)}{3!}\sigma^3 \int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} u^3 e^{-\frac{u^2}{2}} du$$

$$\left. + \frac{n(n-1)(n-2)(n-3)}{4!}\sigma^4 \int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} u^4 e^{-\frac{u^2}{2}} du + \ldots \right]$$

First integral

$$\frac{1}{\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} \int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du = \frac{1}{\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} . \left(1-\Phi\left(-\frac{1}{\sigma}\right)\right) = 1$$

Second integral

$$\frac{n\sigma}{\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} \int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} u e^{-\frac{u^2}{2}} du, \qquad let\ p = \frac{u^2}{2} => du = dp, \quad \frac{1}{2\sigma^2} < p < \infty$$

$$=> \frac{n\sigma}{\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} \int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} u e^{-\frac{u^2}{2}} du = \frac{n\sigma}{\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} \int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-p} dp = \frac{n\sigma}{\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}}$$

For the third integral

$$\frac{n(n-1)\sigma^2}{2!\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\int_{-\frac{1}{\sigma}}^{\infty}\frac{1}{\sqrt{2\pi}}u^2e^{-\frac{u^2}{2}}du$$

But,

$$\int_{-\frac{1}{\sigma}}^{\infty}\frac{1}{\sqrt{2\pi}}u^2e^{-\frac{u^2}{2}}du=\frac{1}{\sqrt{2\pi}}\int_{-\frac{1}{\sigma}}^{0}u^2e^{-\frac{u^2}{2}}du+\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}u^2e^{-\frac{u^2}{2}}du$$

But

$$\int_{0}^{\infty}u^2e^{-\frac{u^2}{2}}du=\frac{\sqrt{2\pi}}{2}$$

But,

Chi-square definition with one degree of freedom is given by

$$f(y)\,\frac{1}{2^{\frac{1}{2}}\,\sigma\left(\frac{1}{2}\right)}z^{\frac{1}{2}-1}e^{-\frac{y}{2}},\quad x>0$$

Now,

$$\int_{-\frac{1}{\sigma}}^{0}u^2e^{-\frac{u^2}{2}}du=\frac{1}{2}\int_{0}^{\frac{1}{\sigma}}p^{\frac{1}{2}}e^{-\frac{p}{2}}dp=\sqrt{2\pi}\;\;P_r\left[\chi^2_{(1)}\leq\frac{1}{\sigma^2}\right]$$

$$(i.e\;after\;taking\;the\;transformation\;p=u^2,0<p<\frac{1}{\sigma^2})$$

Using integration by parts, we have

$$\frac{1}{2}\int_{0}^{\frac{1}{\sigma}}p^{\frac{1}{2}}e^{-\frac{p}{2}}dp=\frac{1}{2}\left(-\frac{2e^{-\frac{1}{2\sigma^2}}}{\sigma}+\sqrt{2\pi}\,P_r\left[X^2_{(1)}\leq\frac{1}{\sigma^2}\right]\right)$$

=> the third term

$$\frac{n(n-1)\sigma^2}{2!\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\int_{-\frac{1}{\sigma}}^{\infty}\frac{1}{\sqrt{2\pi}}u^2e^{-\frac{u^2}{2}}du=\frac{n(n-1)\sigma^2}{2!\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma}+\frac{\sqrt{2\pi}}{2}\left[1+P_r\left(\chi^2_{(1)}\leq\frac{1}{\sigma^2}\right)\right]\right)$$

For the fourth term

$$\frac{n(n-1)(n-2)}{3!\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\sigma^3 \int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} u^3 e^{-\frac{u^2}{2}} du$$

We carry the transformation $p = \frac{u^2}{2}, => du = \frac{dp}{u}, \ \frac{1}{2\sigma^2} < p < \infty$

$$\int_{-\frac{1}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} u^3 e^{-\frac{u^2}{2}} du = \frac{2}{\sqrt{2\pi}} \int_{\frac{1}{2\sigma^2}}^{\infty} p^2 e^{-p} du$$

using integration by parts,

$$\int_{\frac{1}{2\sigma^2}}^{\infty} p^2 e^{-p} du = -pe^{-p}|_{\frac{1}{2\sigma^2}}^{\infty} - \int_{\frac{1}{2\sigma^2}}^{\infty} e^{-p} du = e^{-\frac{1}{2\sigma^2}}\left[1+\frac{1}{2\sigma^2}\right]$$

Now, combining the three integrals, we have

$$E(Y) = 1 + \frac{n\sigma}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} \ e^{-\frac{1}{2\sigma^2}} + \frac{n(n-1)\sigma^2}{2!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1+P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right]\right) + \frac{2n(n-1)(n-2)\sigma^3}{3!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left[1+\frac{1}{2\sigma^2}\right]e^{-\frac{1}{2\sigma^2}} \quad (19)$$

2.3 Derivation of the variance

$Var(Y) = E(Y^2) - (E(Y))^2$

But,

$$E(Y^2) = \int_0^{\infty} \frac{y^2 \frac{1}{n}.y^{\frac{1}{n}-1} e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} dy$$

$$= \int_0^{\infty} \frac{\frac{1}{n}.y^{\frac{1}{n}+1} e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} dy$$

$let\ u = \frac{y^{\frac{1}{n}}-1}{\sigma} => y = (1+\sigma u)^n, \ \frac{du}{dy} = \frac{1}{\sigma}.\frac{1}{n}y^{\frac{1}{n}-1}\ and\ dy = \frac{n\sigma}{y^{\frac{1}{n}-1}}du\ , \ -\frac{1}{\sigma} < u < \infty$

$$\int_0^\infty \frac{\frac{1}{n}.y^{\frac{1}{n}+1}\, e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}dy = \int_{-\frac{1}{\sigma}}^{\infty} \frac{y^2\, e^{-\frac{u^2}{2}}}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}du = \frac{1}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\int_{-\frac{1}{\sigma}}^{\infty}(1+\sigma u)^{2n}\, e^{-\frac{u^2}{2}}du$$

$$=> E(Y^2)$$

$$= \int_{-\frac{1}{\sigma}}^{\infty} \frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}du + \int_{-\frac{1}{\sigma}}^{\infty} \frac{2n\sigma u e^{-\frac{u^2}{2}}}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}du + \int_{-\frac{1}{\sigma}}^{\infty} \frac{2n(2n-1)\sigma^{2u^2 e^{-\frac{u^2}{2}}}}{2!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}du$$

$$+ \int_{-\frac{1}{\sigma}}^{\infty} \frac{2n(2n-1)(2n-2)\sigma^{3u^3 e^{-\frac{u^2}{2}}}}{3!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}du \qquad (20)$$

Similarly the first integral

$$\int_{-\frac{1}{\sigma}}^{\infty} \frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}du = 1$$

For the second integral

$$\int_{-\frac{1}{\sigma}}^{\infty} \frac{2n\sigma u e^{-\frac{u^2}{2}}}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}du = \frac{2n\sigma u e^{-\frac{u^2}{2}}}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}$$

For the third integral

$$\int_{-\frac{1}{\sigma}}^{\infty} \frac{2n(2n-1)\sigma^{2u^2 e^{-\frac{u^2}{2}}}}{2!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}du = \frac{2n(2n-1)\sigma^2}{2!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1+P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right)$$

For the fourth integral

$$\int_{-\frac{1}{\sigma}}^{\infty} \frac{2n(2n-1)(2n-2)\sigma^{3u^3 e^{-\frac{u^2}{2}}}}{3!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}du = \frac{2(2n)(2n-1)(2n-2)\sigma^3}{3!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left[1+\frac{1}{2\sigma^2}\right]e^{-\frac{1}{2\sigma^2}}$$

By combining the four terms of the integration, we have

$$\therefore E(Y^2) = 1 + \frac{2n\sigma u e^{-\frac{u^2}{2}}}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} + \frac{2n(2n-1)\sigma^2}{2!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1+P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right)$$
$$+ \frac{2(2n)(2n-1)(2n-2)\sigma^3}{3!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left[1+\frac{1}{2\sigma^2}\right]e^{-\frac{1}{2\sigma^2}}\ , \qquad (21)$$

2.4 Validation of pdf of the n^{th} power transformation using published transformation

(a) Inverse transformation where $n = -1$

$$f(y) = \frac{\left|\frac{1}{-1}\right| . y^{\frac{1}{-1}-1} \, e^{-\frac{1}{2}\left(\frac{y^{-1}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)} = \frac{|-1|. y^{-2} \, e^{-\frac{1}{2}\left(\frac{y^{-1}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}$$

$$=> \ f(y) = \frac{e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{n}}-1}{\sigma}\right)^2}}{y^2\sigma\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}$$

This conforms to Nwosu et al. [7]

(b) Validation of the pdf for $n = \frac{1}{2}$ (ie square root transformation)

Square root transformation where $n = \frac{1}{2}$

$$f(y) = \frac{\frac{1}{\left(\frac{1}{2}\right)} . y^{\frac{1}{\left(\frac{1}{2}\right)}-1} \, e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{\left(\frac{1}{2}\right)}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)} = \frac{2y^2 \, e^{-\frac{1}{2}\left(\frac{y^2-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}$$

This is the same as Otuonye et al. [8] pdf for square root transformation

(c) Inverse square transformation where $n = -2$

$$f(y) = \frac{\left|\frac{1}{-2}\right| . y^{\frac{1}{(-2)}-1} \, e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{(-2)}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)} = \frac{y^{-\frac{3}{2}} \, e^{-\frac{1}{2}\left(\frac{y^{-\frac{1}{2}}-1}{\sigma}\right)^2}}{2\sigma\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}$$

This conforms to Ibeh et al. [9]

(d) Inverse square root Transformation where $n = -\frac{1}{2}$

$$f(y) = \frac{\frac{1}{\left(-\frac{1}{2}\right)} . y^{\frac{1}{\left(-\frac{1}{2}\right)}-1} \, e^{-\frac{1}{2}\left(\frac{y^{\frac{1}{\left(-\frac{1}{2}\right)}}-1}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}$$

This conforms to Ajibade et al. [10].

2.5 Validation of the mean and variance using published transformations

2.5.1 Square root transformation

2.5.1.1 Mean for square root transformation

Substituting for $n = \frac{1}{2}$ in the mean of the n$^{\text{th}}$ power transformation, we have

$$E(Y) = 1 + \frac{\left(\frac{1}{2}\right)\sigma}{\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}\ e^{-\frac{1}{2\sigma^2}} + \frac{\left(\frac{1}{2}\right)\left(\left(-\frac{1}{2}\right) - 1\right)\sigma^2}{2!\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right)$$

$$= 1 + \frac{\sigma}{2\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}\ e^{-\frac{1}{2\sigma^2}} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\sigma^2}{2\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right)$$

$$= 1 + \frac{\sigma}{2\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}\ e^{-\frac{1}{2\sigma^2}} + \frac{\sigma}{8\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{16\sqrt{2\pi}}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right)$$

$$\therefore E(y) = 1 + \frac{5\sigma}{8\sqrt{2\pi}\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}\ e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{16\sqrt{2\pi}}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) \quad (22)$$

using the second approximation to the binomial as used by Otuonye et al. [10], the mean conform to it.

2.5.1.2 Variance of square root transformation

Given

$$E(Y^2) = 1 + \frac{2n\sigma}{\sqrt{2\pi}\left[1 - \Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}} + \frac{2n(2n-1)\sigma^2}{2!\sqrt{2\pi}\left[1 - \Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right)$$

For $n = -\frac{1}{2}$

$$E(Y^2) = 1 + \frac{2(\frac{1}{2})\sigma}{\sqrt{2\pi}\left[1 - \Phi\left(\frac{-1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}} + \frac{2(\frac{1}{2})\left[2\left(\frac{1}{2}\right) - 1\right]\sigma^2}{2!\sqrt{2\pi}\left[1 - \Phi\left(\frac{-1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right)$$

$$= 1 + \frac{\sigma}{\sqrt{2\pi}\left[1 - \Phi\left(\frac{-1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}} + \frac{(1)(1-1)\sigma^2}{2\sqrt{2\pi}\left[\left[1 - \Phi\left(\frac{-1}{\sigma}\right)\right]\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right)$$

$$= 1 + \frac{\sigma}{\sqrt{2\pi}\left[1 - \Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}} + 0$$

$$\therefore E(Y^2) = 1 - \frac{\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}}$$

Hence

$$Var(Y) = E(Y^2) - [E(y)]^2$$
$$= \left[1 + \frac{\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]} e^{-\frac{1}{2\sigma^2}}\right] - \left[1 + \frac{5\sigma}{8\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\ e^{-\frac{1}{2\sigma^2}} - \frac{\sigma^2}{16\sqrt{2\pi}}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \leq\right.\right.\right.\right.$$
$1\sigma 22$ (23)

2.5.2 Inverse transformation

2.5.2.1 Mean for inverse transformation

Substituting for $n = -1$ in the power expression (1) for $E(Y)$ in the square root transformation, we have

$$E(Y) = 1 + \frac{n\sigma}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\ e^{-\frac{1}{2\sigma^2}} + \frac{n(n-1)\sigma^2}{2!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right]\right) + \frac{2n(n-1)(n-2)\sigma^3}{3!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left[1 + \frac{1}{2\sigma^2}\right] e^{-\frac{1}{2\sigma^2}}$$

$$E(Y) = 1 + \frac{(-1)\sigma}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\ e^{-\frac{1}{2\sigma^2}} + \frac{(-1)(-1-1)\sigma^2}{2!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right)\right]\right) + \frac{2(-1)(-1-1)(-1-2)\sigma^3}{3!\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left[1 + \frac{1}{2\sigma^2}\right] e^{-\frac{1}{2\sigma^2}}$$

$$= 1 - \frac{\sigma}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\ e^{-\frac{1}{2\sigma^2}} - \frac{\sigma}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} e^{-\frac{1}{2\sigma^2}} + \frac{\sigma^2}{2\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(1 + P_r\left[\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right]\right) - \frac{2\sigma^3}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left[1 + \frac{1}{2\sigma^2}\right] e^{-\frac{1}{2\sigma^2}}$$

$$= 1 - \frac{\sigma}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\ e^{-\frac{1}{2\sigma^2}} - \frac{\sigma}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} e^{-\frac{1}{2\sigma^2}} + \frac{\sigma^2}{2\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(1 + P_r\left[\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right]\right) - \frac{2\sigma^3}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} e^{-\frac{1}{2\sigma^2}} - \frac{\sigma}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} e^{-\frac{1}{2\sigma^2}}$$

$$\therefore E(Y) = 1 + \frac{3\sigma}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\ e^{-\frac{1}{2\sigma^2}} + \frac{\sigma^2}{2\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(1 + P_r\left[\chi^2_{(1)} \leq \frac{1}{\sigma^2}\right]\right) - \frac{2\sigma^3}{\sqrt{2\pi}\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)} e^{-\frac{1}{2\sigma^2}} \quad (24)$$

This conforms to Nwosu et al. [9].

2.5.2.2 Variance of inverse transformation

$$(Y^2) = 1 + \frac{2n\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}} + \frac{2n(2n-1)\sigma^2}{2!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) + \frac{2(2n)(2n-1)(2n-2)\sigma^3}{3!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1+\frac{1}{2\sigma^2}\right)e^{-\frac{1}{2\sigma^2}}$$

Substituting for $n = -1$ in the above n[th] transformation for $E(Y^2)$, we have

$$E(Y^2) = 1 + \frac{2(-1)\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}} + \frac{2(-1)(2(-1)-1)\sigma^2}{2!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) + \frac{2(2(-1))(2(-1)-1)(2(-1)-2)\sigma^3}{3!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1+\frac{1}{2\sigma^2}\right)e^{-\frac{1}{2\sigma^2}}$$

$$= 1 - \frac{2\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}} + \frac{2(1)(3)\sigma^2}{2!\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(-\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{\sqrt{2\pi}}{2}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right) - \frac{8\sigma^3}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left(1+\frac{1}{2\sigma^2}\right)e^{-\frac{1}{2\sigma^2}}$$

$$= 1 - \frac{2\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}} - \frac{3\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\frac{e^{-\frac{1}{2\sigma^2}}}{\sigma} + \frac{3\sigma^2}{2\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right] - \frac{8\sigma^3}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}} - \frac{4\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}}$$

$$\therefore E(Y^2) = 1 - \frac{9\sigma}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}} + \frac{3\sigma^2}{2\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right] - \frac{8\sigma^3}{\sqrt{2\pi}\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}e^{-\frac{1}{2\sigma^2}} \quad (25)$$

Without loss of generality, the subsequent terms in $E(Y^2)$ and $E(Y)$ with the factor $e^{-\frac{1}{2\sigma^2}}$ will decay fast to zero for values of σ

Hence,

$$E(Y^2) = 1 + \frac{3\sigma^2}{2\left[1-\Phi\left(-\frac{1}{\sigma}\right)\right]}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]$$

And

$$\therefore E(Y) = 1 + \frac{\sigma^2}{2\left(1-\Phi\left(-\frac{1}{\sigma}\right)\right)}\left(1 + P_r\left[\chi^2_{(1)} \le \frac{1}{\sigma^2}\right]\right)$$

$$\Rightarrow Var(Y) = E(Y^2) - [E(y)]^2$$

$$= \left[1 + \frac{3\sigma^2}{2\left[1 - \Phi\left(-\frac{1}{\sigma}\right)\right]}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right] - \left[1 + \frac{\sigma^2}{2\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}\left(1 + P_r\left[\chi^2_{(1)} \le \frac{1}{\sigma^2}\right]\right)\right]^2$$

$$Var(Y) = \left[\frac{\sigma^2}{2\left[1 - \Phi\left(-\frac{1}{\sigma}\right)\right]}\left[1 + P_r\left(\chi^2_{(1)} \le \frac{1}{\sigma^2}\right)\right]\right] - \left[\frac{\sigma^2}{2\left(1 - \Phi\left(-\frac{1}{\sigma}\right)\right)}\left(1 + P_r\left[\chi^2_{(1)} \le \frac{1}{\sigma^2}\right]\right)\right]^2 \quad (26)$$

This conforms to Nwosu et al. [9].

3 Summary and Conclusion

In this study, the pdf of the n^{th} power transformation of the left-truncated error component of the multiplicative time series model $N(1,\sigma^2)$, was established. Also the mean and variance of the distribution were derived. These results were validated using some published works on power transformations.

Competing Interests

Authors have declared that no competing interests exist.

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