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Self-similar Solutions for a Nonlinear Heat Equation Modelling MEMS

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

This paper deals with the existence and nonexistence of self-similar solutions for a nonlinear heat equation arising from electrostatic MEMS. We show that there exists a critical value A^*, such that if the initial data is less than A^*, then there is no global forward self-similar radial solution. While if the initial data is greater than A^*, then there exists a family of increasing global forward self-similar radial solutions, which goes to ∞ as $r \to \infty$. We also establish the optimal growth rate of these solutions. At last, we give the nonexistence result of backward self-similar solutions.

Keywords: Forward self-similar solutions; backward self-similar solutions; heat equation; MEMS.

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1 INTRODUCTION

The purpose of this paper is to investigate the self-similar solutions for the following nonlinear heat equation

$$\frac{\partial u}{\partial t} = \Delta u + u^{-q}, \qquad (x,t) \in \mathbb{R}^N \times \mathbb{R}^+, \quad (1.1)$$

where $q > 0$ is a constant.

This model (1.1) (with $q = 2$) was first introduced by [1], which models a simple electrostatic Micro-Electro-Mechanical-System (MEMS) device. MEMS devices are key components of many commercial systems, including accelerometers for airbag deployment in automobiles, ink jet printer heads, and chemical sensors. The simplicity and importance of this technique have led many applied mathematicians and engineers to study mathematical models of electrostatic-elastic interactions [2, 3, 4, 5]. The study of self-similar solutions often plays an important role in the investigations of regularity theory, asymptotic stability and singularities of nonlinear problems with similar scaling properties [6, 7, 8], such as the harmonic map heat flow [9], semilinear heat equations [10, 11], etc.

The topic on the self-similar solution has attracted a lot of attention. For example, Brezis and Friedman [12] studied the existence of self-similar solution for the following heat equation with absorption

$$\frac{\partial u}{\partial t} = \Delta u - u^q.$$

Who discovered that

a) when $q \geq 1 + \frac{2}{n}$, the equation admits no singular self-similar solution;

b) when $1 < q < 1 + \frac{2}{n}$, the equation admits a unique singular self-similar solution satisfying $u(x,0) = C\delta(x)$ for any $M > 0$, i.e.

$$\lim_{t \to 0} \int_{|x|<\varepsilon} u(x,t)dx = M, \ \forall \varepsilon > 0.$$

Thereafter, Peletier, Terman [13] studied the self-similar singular solution of the porous medium equation, and Chen, Qi, Wang [14] considered the p-Laplace equation with absorption, in these papers, some singular or very singular self-similar solutions are found. Besides these works, there are also some researches are concerned with the diffusion equation with source u^q, for more details, please refer to [15, 16, 10]. But as far as I know, no researches are concerned with the self-similar solutions of the equation (1.1) or equations with this kind of source u^{-q}. It is worth noting that for this kind of source u^{-q} we considered, there is no singular self-similar solution, in fact, form Theorem 1.2, one will see that the self-similar solutions go to $|x|^{\frac{2}{q+1}}$ as $t \to 0$.

In this paper, we investigate the self-similar solution of the equation (1.1). It is not difficult to see that (1.1) is invariant under the scaling

$$u_\lambda(x,t) = \lambda^{-\alpha} u(\lambda^\beta x, \lambda t),$$

where $\alpha = \frac{1}{q+1}$, $\beta = \frac{1}{2}$. Specially, if we take $\lambda = \frac{1}{t}$, then

$$u(x,t) = t^\alpha u(t^{-\beta}x, 1) = t^\alpha \varphi(t^{-\beta}x), \qquad (1.2)$$

this kind of solution is called forward self-similar solution, here φ satisfies

$$\alpha\varphi(x) - \beta x \cdot \nabla\varphi = \Delta\varphi + \varphi^{-q}, \ x \in \mathbb{R}^N. \quad (1.3)$$

We will look for spherically-symmetric solutions, that is, let $\varphi(|x|) = \varphi(x)$, then the equation (1.3) is transformed into

$$\alpha\varphi(r) - \beta r\varphi'(r) = \varphi'' + \frac{N-1}{r}\varphi' + \varphi^{-q}, \ (1.4)$$

where $\alpha = \frac{1}{q+1}$, $\beta = \frac{1}{2}$. It is natural to assume that $\varphi'(0) = 0$ since φ is radially symmetric. We consider the initial value problem of (1.4) as follows,

$$\varphi'(0) = 0, \varphi(0) = A, \qquad (1.5)$$

where $A > 0$ is a constant.

If $\alpha A^{q+1} = 1$, then A is a solution of (1.4)–(1.5). In what follows, we study the existence of nontrivial global solutions of (1.4)–(1.5).

We have the following results.

Theorem 1.1. *If A satisfies $\alpha A^{q+1} < 1$, then the problem* (1.4)–(1.5) *doesn't admit global solution, and the local solutions $\varphi(r)$ decrease to* 0 *at a finite position.*

Theorem 1.2. *If $\alpha A^{q+1} > 1$, then the problem* (1.4)–(1.5) *admit a global increasing solution for every A with $\alpha A^{q+1} > 1$, and the solution $\varphi(r) \to$*

∞ as $r \to \infty$. Also, we have that the solutions $\varphi(r, A)$ are increasing on A, that is if $A_1 > A_2$ with $\alpha A_i^{q+1} > 1$ ($i = 1, 2$), and $\varphi_i(r, A_i)$ are the corresponding solutions of the problem (1.4)–(1.5)*. Then $\varphi_1'(r) > \varphi_2'(r)$, $\varphi_1(r) > \varphi_2(r)$ for any $r > 0$.*

Furthermore, we have the asymptotic property, that is, there exists a constant $C_0 > 0$ such that

$$\lim_{r\to\infty} r^{-\frac{\alpha}{\beta}}\varphi(r) = C_0.$$

It means that for such initial datum

$$u(x, 0) = \lim_{t\to 0^+} t^{\alpha}\varphi(t^{-\beta}|x|) \sim |x|^{\frac{\alpha}{\beta}},$$

the solution goes to ∞ with speed t^α as $t \to \infty$, more precisely, we have

$$\lim_{t\to\infty} t^{-\alpha}u(x, t) = A, \quad \textit{for any } x \in \mathbb{R}^N.$$

We also consider the backward self-similar solutions of (1.1), that is we investigate the solution of this form

$$u(x, t) = (-t)^{\alpha}\varphi((-t)^{-\beta}x),$$

and we have the following theorem.

Theorem 1.3. *The equation* (1.1) *doesn't admit backward self-similar solution.*

2 NONEXISTENCE OF GLOBAL SOLUTIONS

In what follows, we always assume that the problem (1.4)–(1.5) admit a local classical solution for any given $A > 0$. In fact, the existence of local classical solutions for the problem (1.4)–(1.5) is easy to be obtained by a fixed point approach, we omit the proof.

In this section, we consider the case $\alpha A^{q+1} < 1$. We have the following lemma

Lemma 2.1. *Assume that $\alpha A^{q+1} < 1$, and $\varphi(r)$ is a local classical solution of the problem* (1.4)–(1.5)*. Then $\varphi'(r) < 0$ if $\varphi(r) > 0$.*

Proof. By (1.4)–(1.5) and using L'Hospital's rule, we have

$$N\varphi''(0) = \alpha A - A^{-q} < 0, \tag{2.1}$$

which means that $\varphi'(r) < 0$ for small $r > 0$. In what follows, we show that $\varphi'(r) < 0$ if $\varphi(r) > 0$. Suppose to the contrary, let $r_0 > 0$ be the first zero point of $\varphi'(r)$, and $\varphi(r) > 0$ for any $r \le r_0$. Then $\varphi(r_0) < A$, $\varphi''(r_0) \ge 0$. While by equation (1.4), we see that

$$\varphi''(r_0) = \alpha\varphi(r_0) - \varphi^{-q}(r_0) < 0.$$

It is a contradiction.

Lemma 2.2. *Assume that $\alpha A^{q+1} < 1$, and $\varphi(r)$ is a solution of the problem* (1.4)–(1.5)*. Then there exists $r_0 > 0$ such that $\varphi(r_0) = 0$, and the solution can not exists globally.*

Proof. By Lemma 2.1, we see that φ decreases strictly. We claim that there exists a constant $r_1 > 0$ such that

$$(\alpha + \beta N)\varphi^{q+1}(r_1) = \frac{1}{2}. \tag{2.2}$$

Otherwise, there exists δ with $\frac{\alpha+\beta N}{\alpha} > (\alpha + \beta N)\delta^{q+1} \ge \frac{1}{2}$, such that $\varphi(r) > \delta$ for any $r > 0$. Then there exists a constant $\delta^* \ge \delta$ and $\alpha\delta^{*q+1} < 1$, such that $\varphi(r) \to \delta^*$ as $r \to +\infty$. By equation (1.4), we have

$$\alpha\delta^{*q+1} = 1.$$

It is a contradiction. By (1.4), we see that

$$(r^{N-1}\varphi' + \beta r^N\varphi)' = (\alpha + \beta N)r^{N-1}\varphi - r^{N-1}\varphi^{-q}.$$

Integrating the above equality from r_1 to r gives

$$\begin{aligned} & r^{N-1}\varphi'(r) + \beta r^N\varphi(r) \\ = & \int_{r_1}^{r} ((\alpha + \beta N)\varphi^{q+1} - 1)s^{N-1}\varphi^{-q} ds + c_0 \\ \le & -\frac{1}{2}\int_{r_1}^{r} s^{N-1}\varphi^{-q} ds + c_0 \\ \le & -\frac{1}{2}\varphi^{-q}(r_1)\frac{r^N - r_1^N}{N} + c_0, \end{aligned}$$

which implies

$$\varphi'(r) \le -\frac{1}{2N}\varphi^{-q}(r_1)(r - r_1^N r^{1-N}) + c_0 r^{1-N} - \beta r\varphi(r),$$

and we further have

$$\varphi(r) \le \varphi(r_1) - \frac{1}{2N}\varphi^{-q}(r_1)\int_{r_1}^{r}(s - r_1^N s^{1-N})ds + \int_{r_1}^{r} c_0 s^{1-N} ds - \int_{r_1}^{r} \beta s\varphi(s)ds.$$

Clearly, we have

$$\varphi(r) < 0,$$

when r is appropriately large, which means that there exists r_0 such that $\varphi(r_0) = 0$, and the solution can not exist globally.

Theorem 1.1 is a direct result of Lemma 2.1 and Lemma 2.2.

3 EXISTENCE OF GLOBAL SOLUTIONS AND LARGE TIME BEHAVIOR OF FORWARD SELF-SIMILAR SOLUTIONS

In this section, we consider the case $\alpha A^{q+1} > 1$. We first give the following lemma

Lemma 3.1. *Assume that $\alpha A^{q+1} > 1$, and $\varphi(r)$ is a classical solution of the problem* (1.4)–(1.5)*. Then $\varphi'(r) > 0$ for any $r > 0$.*

Proof. Similar to Lemma 2.1, we have

$$N\varphi''(0) = \alpha A - A^{-q} > 0,$$

which means that $\varphi'(r) > 0$ for small $r > 0$. In what follows, we show that $\varphi'(r) > 0$ for any $r > 0$. Suppose to the contrary, let $r_0 > 0$ be the first zero point of $\varphi'(r)$, then $\varphi''(r_0) \le 0$. By equation (1.4), we also have

$$\varphi''(r_0) = \alpha\varphi(r_0) - \varphi^{-q}(r_0) > 0.$$

It is a contradiction.

We also have the following comparison Lemma

Lemma 3.2. *Assume that $A_1 > A_2$ with $\alpha A_i^{q+1} > 1$ ($i = 1, 2$), and $\varphi_i(r, A_i)$ are the corresponding solutions of the problem* (1.4)–(1.5)*. Then $\varphi_1'(r) > \varphi_2'(r)$, $\varphi_1(r) > \varphi_2(r)$ for any $r > 0$.*

Proof. By the proof of Lemma 3.1, we see that

$$N\varphi_1''(0) = \alpha A_1 - A_1^{-q} > \alpha A_2 - A_2^{-q} = N\varphi_2''(0),$$

which means that $\varphi_1'(r) > \varphi_2'(r)$ for small $r > 0$. Furthermore, we have $\varphi_1'(r) > \varphi_2'(r)$ for any $r > 0$. In fact, otherwise, there exists $r_0 > 0$ such that

$$\varphi_1'(r) > \varphi_2'(r), \text{for } r < r_0, \varphi_1'(r_0) = \varphi_2'(r_0),$$

which means $\varphi_1''(r_0) \le \varphi_2''(r_0)$ and $\varphi_1(r_0) > \varphi_2(r_0)$, while by the equation (1.4), we have

$$\varphi_1''(r_0) - \varphi_2''(r_0) = \big(\alpha\varphi_1(r_0) - \varphi_1^{-q}(r_0)\big) - \big(\alpha\varphi_2(r_0) - \varphi_2^{-q}(r_0)\big) > 0.$$

It is a contradiction.

By Lemma 3.1, we further have

Lemma 3.3. *Assume that $\alpha A^{q+1} > 1$, and $\varphi(r)$ is a local classical solution of the problem* (1.4)–(1.5)*. Then the solution $\varphi(r)$ exists globally and $\varphi(r) \to \infty$ as $r \to \infty$. Furthermore, we show that there exists a constant $C_0 > 0$ such that*

$$\lim_{r\to\infty} r^{-\frac{\alpha}{\beta}}\varphi(r) = C_0.$$

Proof. Let

$$H(r) = M\varphi - \beta r\varphi'$$

with $M = \max\{\alpha, \beta(N-2)\}$. Then we have

$$H'(r) + \beta rH = \beta(M-\alpha)r\varphi + (M - \beta(N-2))\varphi' + \beta r\varphi^{-q} \geq 0.$$

Note that $H(0) = MA > 0$, then

$$H(r) > 0$$

for any $r > 0$, which means

$$(r^{-\frac{M}{\beta}}\varphi)' < 0,$$

and we further have

$$\varphi < c_0 r^{\frac{M}{\beta}}, \quad r\varphi' < \frac{M}{\beta}\varphi, \tag{3.1}$$

for $r > \delta_0 > 0$, where $\delta_0, c_0 > 0$ are constants. Namely, the solution exists globally.

Next, we show $\varphi(r) \to \infty$ as $r \to \infty$. Suppose to the contrary, there exists $C > A$ such that $\varphi(r) \leq C$ for any $r > 0$. Since φ is increasing on r, then $\varphi(r) \to C^* \leq C$ as $r \to \infty$. By equation (1.4), we have $\alpha C^* = C^{*-q}$, it is a contradiction.

In what follows, we turn our attention to the growth rate of the solution. By (1.4), we see that

$$(r^{N-1}\varphi' + \beta r^N\varphi)' = (\alpha + \beta N)r^{N-1}\varphi - r^{N-1}\varphi^{-q}.$$

Integrating the above equality from r_1 to r gives

$$r^{N-1}\varphi'(r) + \beta r^N\varphi(r) = \int_{r_1}^{r}((\alpha+\beta N) - \varphi^{-q-1})s^{N-1}\varphi ds + r_1^{N-1}\varphi'(r_1) + \beta r_1^N\varphi(r_1),$$

and we further have

$$\frac{\varphi'(r)}{r\varphi} + \beta = \frac{\int_{r_1}^{r}((\alpha+\beta N) - \varphi^{-q-1})\varphi s^{N-1}ds}{r^N\varphi} + \frac{r_1^{N-1}\varphi'(r_1) + \beta r_1^N\varphi(r_1)}{r^N\varphi},$$

by (3.1), and let $r \to \infty$, we obtain

$$\begin{aligned}
\beta &= \lim_{r\to\infty} \frac{\int_{r_1}^{r}((\alpha+\beta N)\varphi^{q+1} - 1)s^{N-1}\varphi^{-q}ds}{r^N\varphi} \\
&= \lim_{r\to\infty} \frac{((\alpha+\beta N)\varphi^{q+1} - 1)r^{N-1}\varphi^{-q}}{r^N\varphi' + Nr^{N-1}\varphi} \\
&= \lim_{r\to\infty} \frac{(\alpha+\beta N)r^{N-1}\varphi}{r^N\varphi' + Nr^{N-1}\varphi} \\
&= \lim_{r\to\infty} \frac{\alpha+\beta N}{\frac{r\varphi'}{\varphi} + N},
\end{aligned}$$

which means that

$$\lim_{r\to\infty} \frac{r\varphi'}{\varphi} = \frac{\alpha}{\beta}.$$

Then for any sufficiently small $\varepsilon > 0$, there exists $M > 0$, such that

$$(\frac{\alpha}{\beta} - \varepsilon)\varphi < r\varphi' < (\frac{\alpha}{\beta} + \varepsilon)\varphi, \quad \text{for any } r > M,$$

and we further obtain that there exist two constants $C_1 = C_1(M) > 0, C_2 = C_2(M) > 0$, such that

$$C_1 r^{\frac{\alpha}{\beta} - \varepsilon} < \varphi(r) < C_2 r^{\frac{\alpha}{\beta} + \varepsilon}, \forall\, r > M. \tag{3.2}$$

Next, we show

$$\varphi(r) \sim r^{\frac{\alpha}{\beta}}.$$

Multiplying the equation (1.4) by $r^{-\frac{\alpha}{\beta}-1}$, we obtain

$$\beta(r^{-\frac{\alpha}{\beta}}\varphi)' = -r^{-\frac{\alpha}{\beta}-N}(r^{N-1}\varphi')' - \varphi^{-q}r^{-\frac{\alpha}{\beta}-1}.$$

Integrating this equality from r_0 to r, gives

$$\begin{aligned}
&\beta r^{-\frac{\alpha}{\beta}}\varphi(r) = \beta r_0^{-\frac{\alpha}{\beta}}\varphi(r_0) - \int_{r_0}^{r} s^{-\frac{\alpha}{\beta}-N}(s^{N-1}\varphi')'ds - \int_{r_0}^{r}\varphi^{-q}s^{-\frac{\alpha}{\beta}-1}ds \\
=&\beta r_0^{-\frac{\alpha}{\beta}}\varphi(r_0) - r^{-\frac{\alpha}{\beta}-1}\varphi'(r) + r_0^{-\frac{\alpha}{\beta}-1}\varphi'(r_0) - (\frac{\alpha}{\beta}+N)\int_{r_0}^{r} s^{-\frac{\alpha}{\beta}-2}\varphi'(s)ds - \int_{r_0}^{r}\varphi^{-q}s^{-\frac{\alpha}{\beta}-1}ds \\
=&\beta r_0^{-\frac{\alpha}{\beta}}\varphi(r_0) - r^{-\frac{\alpha}{\beta}-1}\varphi'(r) + r_0^{-\frac{\alpha}{\beta}-1}\varphi'(r_0) - (\frac{\alpha}{\beta}+N)r^{-\frac{\alpha}{\beta}-2}\varphi(r) + (\frac{\alpha}{\beta}+N)r_0^{-\frac{\alpha}{\beta}-2}\varphi(r_0) \\
&-(\frac{\alpha}{\beta}+N)(\frac{\alpha}{\beta}+2)\int_{r_0}^{r} s^{-\frac{\alpha}{\beta}-3}\varphi(s)ds - \int_{r_0}^{r}\varphi^{-q}s^{-\frac{\alpha}{\beta}-1}ds,
\end{aligned} \tag{3.3}$$

where $r_0 > 0$ is a sufficiently large constant, which is to be determined. By (3.1) and (3.2), we see that

$$\lim_{r\to+\infty} r^{-\frac{\alpha}{\beta}-1}\varphi'(r) = 0, \quad \lim_{r\to+\infty} r^{-\frac{\alpha}{\beta}-2}\varphi(r) = 0. \tag{3.4}$$

Next, we estimate the last two terms of (3.3). By (3.2), we see that for any $\varepsilon < \min\{\frac{1}{2}, \frac{\alpha}{2\beta}\}$, when $r_0 > M$,

$$\int_{r_0}^{\infty} s^{-\frac{\alpha}{\beta}-3}\varphi(s)ds \le C_2\int_{r_0}^{\infty} s^{\varepsilon-3}ds < C_2 r_0^{\varepsilon-2}, \tag{3.5}$$

$$\int_{r_0}^{\infty}\varphi^{-q}s^{-\frac{\alpha}{\beta}-1}ds \le C_1^{-q}\int_{r_0}^{\infty} s^{-\frac{\alpha}{\beta}(q+1)-\varepsilon q-1}ds < \frac{C_1^{-q}}{\frac{\alpha}{\beta}(q+1)}r_0^{-\frac{\alpha}{\beta}(q+1)-\varepsilon q}. \tag{3.6}$$

Substituting (3.4)-(3.6) into (3.3), and letting $r \to \infty$, we obtain, for any $r_0 > M$,

$$\begin{aligned}
&\lim_{r\to\infty}\beta r^{-\frac{\alpha}{\beta}}\varphi(r) = \beta r_0^{-\frac{\alpha}{\beta}}\varphi(r_0) + r_0^{-\frac{\alpha}{\beta}-1}\varphi'(r_0) + (\frac{\alpha}{\beta}+N)r_0^{-\frac{\alpha}{\beta}-2}\varphi(r_0) \\
&-(\frac{\alpha}{\beta}+N)(\frac{\alpha}{\beta}+2)\int_{r_0}^{r} s^{-\frac{\alpha}{\beta}-3}\varphi(s)ds - \int_{r_0}^{r}\varphi^{-q}s^{-\frac{\alpha}{\beta}-1}ds \\
&\ge C_1\beta r_0^{-\varepsilon} - C_3 r_0^{\varepsilon-2} - C_4 r_0^{-\frac{\alpha}{\beta}(q+1)-\varepsilon q}.
\end{aligned}$$

Take r_0 sufficiently large such that

$$C_3 r_0^{\varepsilon-2} + C_4 r_0^{-\frac{\alpha}{\beta}(q+1)-\varepsilon q} \le \frac{1}{2}C_1\beta r_0^{-\varepsilon}.$$

Then we obtain

$$\lim_{r\to\infty} r^{-\frac{\alpha}{\beta}}\varphi(r) \geq \frac{1}{2}C_1 r_0^{-\varepsilon}. \tag{3.7}$$

On the other hand, we also have

$$\lim_{r\to\infty} \beta r^{-\frac{\alpha}{\beta}}\varphi(r) \leq \beta r_0^{-\frac{\alpha}{\beta}}\varphi(r_0)$$

$$+r_0^{-\frac{\alpha}{\beta}-1}\varphi'(r_0) + (\frac{\alpha}{\beta}+N)r_0^{-\frac{\alpha}{\beta}-2}\varphi(r_0). \tag{3.8}$$

By (3.7)–(3.8), we see that there exist positive constant r^*, $\underline{M} < \overline{M}$, such that

$$\underline{M}r^{\frac{\alpha}{\beta}} \leq \varphi(r) \leq \overline{M}r^{\frac{\alpha}{\beta}},\ \forall\ r > r^*. \tag{3.9}$$

We claim that there exists a constant $C_0 > 0$ such that

$$\lim_{r\to\infty} r^{-\frac{\alpha}{\beta}}\varphi(r) = C_0.$$

Otherwise, there exists two sequences $\{\hat{r}_n\}$, $\{\tilde{r}_n\}$ with $\hat{r}_n > \tilde{r}_n \to \infty$ such that

$$\underline{C} = \lim_{n\to\infty} \hat{r}_n^{-\frac{\alpha}{\beta}}\varphi(\hat{r}_n) < \lim_{n\to\infty} \tilde{r}_n^{-\frac{\alpha}{\beta}}\varphi(\tilde{r}_n) = \overline{C}. \tag{3.10}$$

Taking $r = \hat{r}_n$, $r_0 = \tilde{r}_n$ in (3.3), letting $n \to \infty$, and combining (3.4)–(3.6), we obtain

$$\beta\underline{C} = \beta\overline{C} + \lim_{n\to\infty} \tilde{r}_n^{-\frac{\alpha}{\beta}-1}\varphi'(\tilde{r}_n)$$

$$+(\frac{\alpha}{\beta}+N)\tilde{r}_n^{-\frac{\alpha}{\beta}-2}\varphi(\tilde{r}_n) \geq \beta\overline{C}. \tag{3.11}$$

It contradicts with (3.10), the proof is complete.

Theorem 1.2 is a direct result of Lemma 3.1, Lemma 3.2 and Lemma 3.3.

4 NONEXISTENCE OF BACKWARD SELF-SIMILAR SOLUTION

We also consider the backward self-similar solution of the equation (1.1), that is let

$$u(x,t) = (-t)^{\alpha}\varphi((-t)^{-\beta}x),$$

then the equation (1.1) is equivalent to

$$\Delta\varphi - \beta\xi\cdot\nabla\varphi + \alpha\varphi + \varphi^{-q} = 0, \tag{4.1}$$

where $\alpha = \frac{1}{q+1}$, $\beta = \frac{1}{2}$. Multiplying the equation (4.1) by $e^{-\frac{\beta}{2}|\xi|^2}\varphi(\xi)$, and integrating over $\mathbb{R}^N$, we see that

$$\int_{\mathbb{R}^N} e^{-\frac{\beta}{2}|\xi|^2}|\nabla\varphi(\xi)|^2 dx$$

$$+\int_{\mathbb{R}^N} e^{-\frac{\beta}{2}|\xi|^2}(\alpha\varphi^2 + \varphi^{1-q})dx = 0.$$

which means that (4.1) doesn't admit solution. Theorem (1.3) is proved.

5 CONCLUSION AND PROSPECT

In this paper, we establish the existence of forward self-similar solutions, and we also give the optimal growth rate of these solutions, which implies that the self-similar solutions go to $|x|^{\frac{2}{q+1}}$ as $t \to 0$, so there is no singular self-similar solution for the equation (1.1). At Section 4, we also discussed the existence of backward self-similar solution, and a nonexistence result is given. In the future, we will continue to study the self-similar solutions of some degenerate or singular parabolic equations, and the stability of self-similar solutions will be discussed.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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