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Outage Analysis of a Multi-User Spatial Diversity System in a Shadow-Fade Propagating Channel

Vahid Emamian[1*]

[1]*Engineering Department, St. Mary's University, One Camino Santa Mario, San Antonio, Texas, 78228, United States.*

Author's contribution

This work was carried out by author VE. He designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. He has read and approved the final manuscript.

Research Article

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ABSTRACT

In a wireless network, communication between a source and a destination mobile station (DMS) fails to establish if the source or the DMS is located inside a deep shadow-fading propagating channel. In this situation, intermediate mobile stations may be used to relay the signal between the two nodes. In a cellular system the source is a base station (BS) and the DMS is a weak mobile station (MS) while in an ad-hoc network, the source and the DMS are two nodes of the network. This paper presents the scheme of "multi-user spatial diversity" as a method of diversity to combat the undesired shadow-fade channel behavior. A model is presented for the case where one or several mobile stations (MSs) relay the signal between the source and the DMS, in a shadow-fading environment. A formula is derived for the average outage probability of the received signal-to-noise ratio at the DMS, when M intermediate MSs relay the signal from the source to the DMS according to a particular protocol. The outage probability improves as the number of the relays increases.

Keywords: Multi-user spatial diversity; Ad-hoc networks; wireless cellular communications; outage probability; independent shadowing; relay mobile station; macro diversity.

**Corresponding author: E-mail: vemamian@stmarytx.edu;*

1. INTRODUCTION

Spatial diversity (using multiple antennas at the transmitter and/or receiver) is a common method of combating the deleterious effects of fading and shadowing in a wireless communication network [1-2]. This method, however, has usually been associated with point-to-point links. Recently the work of [3-7] has suggested the idea of using a possible idle mobile station (MS) as a relay between a source and a destination mobile station (DMS) to envisage a new type of (mobile-assisted) spatial diversity that may be classified as distributed spatial diversity 5].

Some form of collaboration among users is needed in order to implement distributed spatial diversity. In 6] the users select one partner from the in-cell users. Each user decodes the information symbols of its partner and transmits them along with its own information. Consequently, the user achieves spatial diversity by using its partner's antenna. It turns out that even though the inter-user channel is noisy, the set of achievable rates for a system with two cooperative users is a superset of the capacity region obtained without cooperation. The idea of collaboration among MSs in a cellular system is proposed in 3,5]. In the scheme that is discussed in these references, an MS retransmits a signal from the base station (BS) to a nearby MS that is in a shadowed and faded region relative to the BS. In 3], the concept of shadow-fade environment is described in details. In the analysis provided in [3,5] the additional noise at the relay was neglected and the gain of the amplifier at the relay was independent of the received signal's power. Theoretical and simulation results in [3,5] show substantial reduction of the outage probability for a collaborative system, in comparison with that of a non-collaborative system using 2-transmit antenna micro-diversity in a composite lognormal-Rayleigh shadowing-fading environment.

The work in2,6,7,8] analyzes the performance of a system with a relay that decodes and forwards its partner's information symbols versus a system with a relay that only amplifies the received signal. For independent Gaussian flat fading channels with path loss, it is argued in [7], by simulations, that the less complex cooperative system with an amplifying relay has a similar average bit error rate as the system using a decoding relay. To confirm the good performance of the amplify-and-forward strategy, the outage probability of the achievable rate in a two steps collaborative system using orthogonal channels (e.g., an Frequency Division Multiple Access or FDMA system) is computed in [5]. For an amplifying relay, it is shown that this outage probability is comparable to the outage probability of the same system that is optimized to select among: no relay, amplifying relay, and decoding relay, depending on the reliability of the connection between source and relay. Furthermore, it is argued in [8] that the amplify-and-forward strategy slightly loses performance from the case of the ideal transmit diversity in which the 2 transmit antennas are located at the source (i.e., noiseless communication between source and relay). Related studies to this work are further presented in [9-19].

The system presented in this paper is depicted inFig. **1**. When the communication between the source and the DMS is affected by a deep shadow-fade situation, the source is allowed to use other MSs as relays, to communicate with the DMS. Let's call an intermediate MS a relay MS or RMS. Since the relay channels are often independent, multiple independently faded-shadowed versions of the transmitted signal are received, thereby providing diversity transmission.

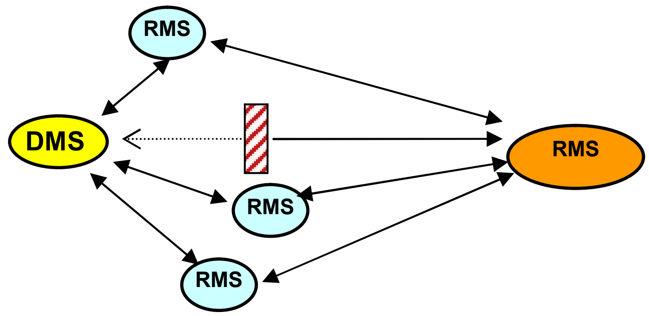

Fig. 1. Multi-user spatial diversity in a wireless network. Nearby MSs (RMSs) are employed to relay the signal to the destination mobile station (DMS)

Because usually more than one MS can be involved in relaying the signal, let's call such diversity "multi-user spatial diversity". The major contributions of the paper are as follows:

1. A formula is derived for the outage probability of the received signal-to-noise ratio (SNR) at the DMS, when M RMSs are used as relays between the source and the DMS, assuming the receiver at the DMS uses selection combining and the channels incur path loss and independent multipath flat fading-lognormal shadowing.
2. A complete discussion of multi-user spatial diversity and the associated signal models in a wireless network are presented.
3. The distribution of the received Signal to Noise Ratio (SNR) and the outage probability at DMS are derived, when there is a constraint on the transmit power at all relays (i.e. they are equipped with automatic gain control (AGC)). The reason for using AGCs is that the relays are indeed MSs with limited power. This power is used for the sake of other MSs, and one may not want to share more than a specific amount of its power for relaying purposes.
4. The paper also presents results for the case in which the additional noise in the relay is ignored. This yields a simpler closed-form formula for the outage probability of the received SNR. The paper also includes the results of a study into the probability of restoring communication between the source and the DMS as a function of the number of RMSs.

2. MULTI-USER SPATIAL DIVERSITY

Let's define user diversity to be a form of spatial diversity in which a signal is relayed, through an independent channel, between the source and the DMS by another MS in the network. In the method analyzed in this paper, multiple users are involved in relaying the signal between the source and the DMS, leading to the use of the term "multi-user spatial diversity". Only amplification at the relays is considered in this paper.

Fig. 2 shows a system that employs such multi-user spatial diversity, and the channel designation for the various links. In this figure RMS_1, RMS_2, …., RMS_M are M relays *nearby to theDMS*, assumed to be ordered with RMS_1 the nearest MS to the DMS and RMS_2 the second nearest MS to the DMS and so on. When the direct channel from the source to the DMS faces a deep shadow-fade situation, the communication is recovered by employing a relay (the nearest MS i.e. RMS_1) between them. If the problem still remains, two relays (the nearest plus the second nearest i.e. RMS_1 and RMS_2) are employed to reestablish the connection and so on. The source and the DMS are assumed to be communicating while M

MSs relay the signal between them. Assuming that C_0, C_1, ..$C_M$are$M$+1 orthogonal channels, the scheme works as follows. The source broadcasts the signal to the DMS and the RMSs in channel C_0. RMS_m (m=1,2, …, M) converts the data from the channel C_0 to channel C_m, and then amplifies and relays it to the DMS. Therefore, the DMS receives signals from M+1 orthogonal channels (C_0, C_1, ... C_M).

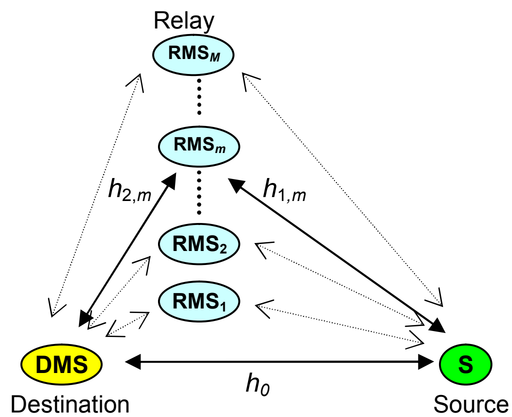

Fig. 2. *M* RMSs relay the signal between the source and the DMS.

The discrete-time baseband equivalent multichannel model consists of M +1 sub channels between the source and the DMS

$$\begin{aligned} y_0 &= h_0\sqrt{\varepsilon_0}x + z_0 \\ y_m &= h_{2,m}\alpha_m\left(h_{1,m}\sqrt{\varepsilon_0}x + z_{1,m}\right) + z_{2,m}, \qquad m \in [1, M], \end{aligned} \tag{1}$$

Where$x$ is the transmitted signal from the source, y_0 is the received signal from the direct path (source → DMS), and y_m is the received signal from the m^{th} relay (RMS_m→ DMS). The maximum transmit power from the source is given by ε_0. If automatic gain control (AGC) is used at the mth relay, α_m is an amplification factor that ensures the transmit power at the RMS_m falls below a specific value of ε_m. z_0 and $z_{2,m}$ are received AWGN (Additive White Gaussian Noise) , both assumed to have power N_0. h_0, $h_{1,m}$, and $h_{2,m}$ are assumed to be independent and complex Gaussian distributed with zero mean and variance Ω_0, $\Omega_{1,m}$, and $\Omega_{2,m}$ respectively. Also $z_{1,m}$ is the additional noise at the mth relay.

The parameters Ω_0, $\Omega_{1,m}$, and $\Omega_{2,m}$ are assumed to be lognormal with the following PDF (probability density function)

$$f_{\Omega_0}(\Omega)=\frac{1}{\sqrt{2\pi}\sigma_0\Omega}e^{-\frac{(\ln(\Omega)-\mu_0(L))^2}{2\sigma_0^2}},\ f_{\Omega_{1,m}}(\Omega)=\frac{1}{\sqrt{2\pi}\sigma_{1,m}\Omega}e^{-\frac{(\ln(\Omega)-\mu_{1,m}(d_m))^2}{2\sigma_{1,m}^2}},$$

$$f_{\Omega_{2,m}}(\Omega)=\frac{1}{\sqrt{2\pi}\sigma_{2,m}\Omega}e^{-\frac{(\ln(\Omega)-\mu_{2,m}(r_m))^2}{2\sigma_{2,m}^2}} \tag{2}$$

where $\mu_0(L)$, $\mu_{1,m}(d_m)$ are the area mean powers for the distances L and d_m of the source to the DMS and RMS$_m$ respectively, and $\mu_{2,m}(r_m)$ is the area mean power for the distance r_m of the RMS$_m$ to the DMS (
Fig. 3). These parameters are given by

$$\mu_0(L)=\mu-\beta\ln(L/d),\ \mu_{1,m}(d_m)=\mu-\beta\ln(d_m/d),\ \mu_{2,m}(r_m)=\mu-\beta\ln(r_m/d),$$

where d, μ, and β are the reference distance, the area mean power at the reference distance, and the path loss exponent respectively.

In the following sections the outage probabilities of systems employing multi-user spatial diversity, as defined in this section, are investigated for flat shadow-fading channels. All the simulations in this paper is performed in MATLAB.

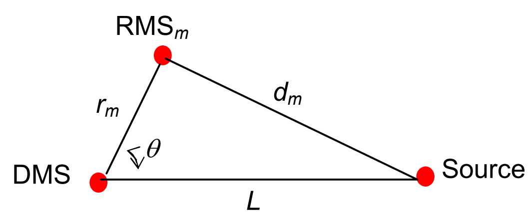

Fig. 3. Relative positions of the source, the DMS and the RMS$_m$

3. OUTAGE PROBABILITIES

The outage probability of the received signal at the DMS, when *M* RMSs relay the signal between the source and the DMS according to the protocol described in the previous section is shown here. Outage happens when the received SNR at the DMS falls below a specific threshold. In the first subsection the assumption is that transmit power at RMS$_m$ is limited to ε_m, and in the second subsection a simpler formula is derived for the outage by ignoring the noise at the relays.

3.1 With AGC at Relays

First the probability of outage is calculated at the DMS for the channel 0 (direct path between source and DMS). Using(1), the signal-to-noise ratio for this channel is given by

$$\gamma_0 = \left|h_0\right|^2 \varepsilon_0 / N_0 \quad (3)$$

Since $\left|h_0\right|$ has a Rayleigh distribution, it can be shown that γ_0 is exponentially distributed with the following PDF

$$f_{\gamma_0}(\gamma \mid \Omega_0) = \frac{1}{\Omega_0} e^{-\frac{\gamma}{\Omega_0}} u(\gamma) . \quad (4)$$

For a given Ω_0, the probability of γ_0 being less than a specific threshold (γ) is given by $P_0(\Omega_0) = 1 - e^{-\frac{\gamma}{\Omega_0}}$. Since Ω_0 is random, the average outage probability is given by

$$P_0 = E_{\Omega_0}[P(\Omega_0)] = \int_0^{\infty} \frac{1 - e^{-\frac{\gamma}{\Omega_0}}}{\sqrt{2\pi}\sigma_0 \Omega} e^{-\frac{(\ln(\Omega) - \mu_0(L))^2}{2\sigma_0{}^2}} d\Omega . \quad (5)$$

Now the probability of outage is calculated at the DMS for channel *m* (source →RMS$_m$→ DMS). Assuming that the additional noise at the m^{th} relay is AWGN with power N_0, the SNR at this relay (RMS$_m$) is

$$\gamma_{1,m} = \left|h_{1,m}\right|^2 \varepsilon_0 / N_0 . \quad (6)$$

Once again, $\gamma_{1,m}$ is exponentially distributed with parameter $\Omega_{1,m}$. To satisfy the transmit power constraint at RMS$_m$, the AGC power gain is[1]

$$\alpha_m^2 = \frac{\varepsilon_m}{\left|h_{1,m}\right|^2 \varepsilon_0 + N_0} . \quad (7)$$

Using (1), the SNR at the DMS is given by

$$\gamma_m = \frac{\left|h_{1,m}\right|^2 \left|h_{2,m}\right|^2 \alpha_m^2 \varepsilon_0}{N_0 \left(1 + \left|h_{2,m}\right|^2 \alpha_m^2\right)} . \quad (8)$$

Substituting (7)in (8), the following is obtained

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[1] *The transmit power at the RMS$_m$is ε_m . On the other hand from (1) this power is calculated to be. Equation (7) is obtained by equalizing these two.*

$$\gamma_m = \frac{\gamma_{1,m}\gamma_{2,m}}{\gamma_{1,m} + \gamma_{2,m} + 1}, \tag{9}$$

where

$$\gamma_{2,m} = \left|h_{2,m}\right|^2 \varepsilon_m \Big/ N_0 . \tag{10}$$

Because $\gamma_{1,m}$ and $\gamma_{2,m}$ are independent, the cumulative distribution function (CDF) of γ_m can be found as

$$\begin{aligned} F_{\gamma_m}(\gamma \,|\, \Omega_{1,m}, \Omega_{2,m}) &= \int_0^\infty P\left(\frac{\gamma_{1,m}\lambda}{\gamma_{1,m} + \lambda + 1} \le \gamma\right) f_{\gamma_{2,m}}(\lambda)\, d\lambda \\ &= \int_0^\gamma P\left(\gamma_{1,m} \ge \frac{\gamma\lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda)\, d\lambda + \int_\gamma^\infty P\left(\gamma_{1,m} \le \frac{\gamma\lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda)\, d\lambda \\ &= I_{1,m} + I_{2,m}, \end{aligned} \tag{11}$$

where

$$I_{1,m} \sqcup \int_0^\gamma P\left(\gamma_{1,m} \ge \frac{\gamma\lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda)\, d\lambda = \int_0^\gamma f_{\gamma_{2,m}}(\lambda)\, d\lambda = 1 - e^{-\frac{\gamma}{\Omega_{2,m}}}, \tag{12}$$

and

$$\begin{aligned} I_{2,m} &= \int_\gamma^\infty P\left(\gamma_{1,m} \le \frac{\gamma\lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda)\, d\lambda = \frac{1}{\Omega_{2,m}} \int_\gamma^\infty \left(1 - e^{-\frac{\gamma\lambda + \gamma}{\Omega_{1,m}(\lambda - \gamma)}}\right) e^{-\frac{\lambda}{\Omega_{2,m}}} d\lambda \\ &= e^{-\frac{\gamma}{\Omega_{2,m}}} - \frac{1}{\Omega_{2,m}} e^{-\gamma\left(\frac{\Omega_{1,m} + \Omega_{2,m}}{\Omega_{1,m}\Omega_{2,m}}\right)} \int_0^\infty e^{-\left(\frac{\gamma^2 + \gamma}{\Omega_{1,m}\lambda} + \frac{\lambda}{\Omega_{2,m}}\right)} d\lambda = e^{-\frac{\gamma}{\Omega_{2,m}}} - 2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m}\Omega_{2,m}}}\, e^{-\gamma\left(\frac{\Omega_{1,m} + \Omega_{2,m}}{\Omega_{1,m}\Omega_{2,m}}\right)} K_1\left(2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m}\Omega_{2,m}}}\right), \end{aligned} \tag{13}$$

where $K_1(.)$ is a modified Bessel function of the second kind. The third equality was obtained by changing the integration variable from λ to $\lambda + \gamma$. From (12) and (13) the following is obtained

$$F_{\gamma_m}(\gamma \,|\, \Omega_{1,m}, \Omega_{2,m}) = 1 - 2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m}\Omega_{2,m}}}\, e^{-\gamma\left(\frac{\Omega_{1,m} + \Omega_{2,m}}{\Omega_{1,m}\Omega_{2,m}}\right)} K_1\left(2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m}\Omega_{2,m}}}\right). \tag{14}$$

To find the average outage probability for path $m \in [1, M]$, let's compute the expected value of (14) with respect to $\Omega_{1,m}$ and $\Omega_{2,m}$, i.e. $E_{\Omega_{1,m}\Omega_{2,m}}\left[F_{\gamma_m}(\gamma \mid \Omega_{1,m}, \Omega_{2,m})\right]$, which is given by

$$P_m(\gamma) = \int_0^{\infty}\int_0^{\infty} F_{\gamma_m}(\gamma \mid \Omega_{1,m}, \Omega_{2,m}) f_{\Omega_{1,m}}(\Omega_{1,m}) f_{\Omega_{2,m}}(\Omega_{2,m}) d\Omega_{1,m} d\Omega_{2,m}. \quad m \in [1, M] \tag{15}$$

As mentioned earlier, the method of "selection combining" is used at the receiver. Therefore the average outage probability at the DMS is the product of the outage probabilities for all the paths 1 , i.e.

$$P(\gamma) = \prod_{m=0}^{M} P_m(\gamma). \tag{16}$$

The distributions of $\Omega_{1,m}$ and $\Omega_{2,m}$ are functions of r_m and d_m respectively. In order to obtain useful and informative results, the expected values of these parameters are used (i.e. $E[r_m]$ and $E[d_m]$) to evaluate $\tilde{P}(\gamma)$ for the average RMS distance. In the appendix $E[r_m]$ and $E[d_m]$ are computed as a function of *m*, which is ordered according to the proximity of the RMSs to the DMS and the number of MSs randomly distributed inside a circle of radius *R* around the DMS.

Fig. 4 shows the outage probability $\tilde{P}(\gamma)$ versus the threshold SNR (γ) for $M \in [0, 3]$. To verify the results, a simulation is conducted. The result of simulation is presented on the figure with broken lines. The parameters of the simulation are listed in Table 1 (as well as Table 2 in the appendix). These parameters are also used for the evaluation of the formulas.

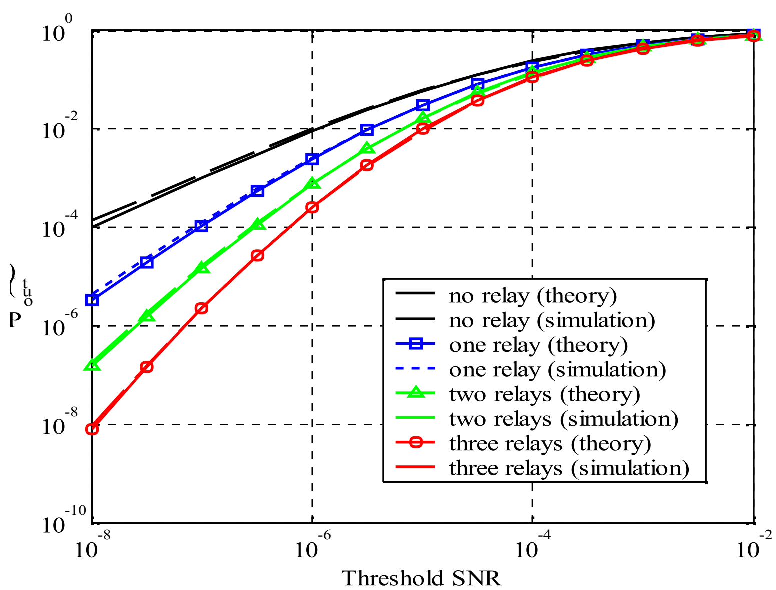

Fig. 4. Outage probability versus threshold SNR for a system with AGC at the relay

Table 1. Parameters of the simulation

Parameter	*L*	*R*	μ	*D*	β	σ_0 , $\sigma_{1,M}$, $\sigma_{2,M}$
VALUE	500 M	100 M	-10 DB	10 M	5	B

3.2 Simplified Formula

Here, a simpler formula than given in section 3 is derived for the outage probability of the received SNR at the DMS, when the noise in the relays is ignored. While this assumption is not realistic, it is shown that the simpler formula provides results that are quite close to that in section 3.

From (1), by ignoring the noise in the relays ($z_{1,m} = 0$), the following formula is obtained

$$\gamma_m = \frac{\left|h_{1,m}\right|^2 \alpha_m^2 \left|h_{2,m}\right|^2 \varepsilon_0}{N_0}.$$

Therefore, $\gamma_m = \frac{\alpha_m^2 N_0}{\varepsilon_m} \gamma_{1,m} \gamma_{2,m}$. For simplicity let's choose $\alpha_m^2 = \varepsilon_m / N_0$, which yields $\gamma_m = \gamma_{1,m} \gamma_{2,m}$ (a product of two exponential R.V.s). It can be shown that the CDF of the γ_m is expressed as[2]

$$F_{\gamma_m}(\gamma \mid \hat{\Omega}_m) = 1 - \frac{2\sqrt{\gamma}}{\hat{\Omega}_m} K_1(\frac{2\sqrt{\gamma}}{\hat{\Omega}_m}) \qquad s > 0,\, \hat{\Omega}_m > 0, \tag{17}$$

where $\hat{\Omega}_m = \sqrt{\Omega_{1,m}\Omega_{2,m}}$ is a lognormal random variable, i.e. $\ln(\hat{\Omega}_m)$ is a normal random variable with $\hat{\mu}_m = (\mu_{1,m}(d_m) + \mu_{2,m}(r_m))/2$ and $\hat{\sigma}_m = (\sigma_{1,m} + \sigma_{2,m})/2$.

Using (17), the probability of outage for channel *m* is given by

$$P_m(\gamma) = \frac{1}{\sqrt{2\pi}\hat{\sigma}_m \hat{\Omega}_m} \int_0^\infty \left(1 - \frac{2\sqrt{\gamma}}{\hat{\Omega}_m} K_1(\frac{2\sqrt{\gamma}}{\hat{\Omega}_m})\right) \exp\left(-\frac{(\ln(\hat{\Omega}_m) - \hat{\mu}_m)^2}{2\hat{\sigma}_m{}^2}\right) d\hat{\Omega}_m, \quad m = 1, 2, ..., M \tag{18}$$

This equation involves a single integral, to be contrasted with the double integral produced in the calculation of section 3. The outage happens when received SNR from all *M*+1 channels (channel 0 plus *M* relay channels) fall below a specific threshold. Assuming that all *M*+1 channels are independent, the outage is given by (16).

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[2]*This can be obtained as follows:* $F_{\gamma_m}(\gamma \mid \Omega_1, \Omega_2) = \Pr\{\gamma_{1,m}\gamma_{2,m} \le \gamma\} = \int_0^\infty \Pr\{\gamma_{1,m} \le \frac{\gamma}{\lambda} \mid \gamma_{2,m} = \lambda\} f_{\gamma_{2,m}}(\lambda) d\lambda =$

$\int_0^\infty (1 - e^{-\gamma/\lambda\Omega_{1,m}}) \frac{1}{\Omega_2} e^{-\lambda/\Omega_{2,m}} d\lambda = 1 - \frac{2\sqrt{\gamma}}{\sqrt{\Omega_{1,m}\Omega_{2,m}}} K_1(\frac{2\sqrt{\gamma}}{\sqrt{\Omega_{1,m}\Omega_{2,m}}})$.

Generalized version of this proof for χ^2 random variable can be found in 10.

Fig. **5** gives this outage probability for the received SNR versus the threshold SNR for *M*=0,1,2,3, To confirm this result, a simulation has been conducted. The result of simulation is presented on the same figure. The parameters that are used here are listed in Table 1. The figure shows the improvement in outage probability as the number of the relays increases. Since the noise at the relays is neglected, the outage probability is slightly reduced, compared with the one given in section 3.

Now suppose that the source and the DMS fail to communicate due to shadow-fade channel or severity of the noise. It would be interesting to determine the number of relays needed to re-establish communication between the source and the destination. This is assumed to be the minimum number of relays required so that the received SNR at the DMS is above a specific threshold γ_{th}, when the direct path SNR, γ_0, is below this threshold . Since the channels are assumed to be independent, we have

$$\Pr\{\gamma \geq \gamma_{th} | \gamma_0 \leq \gamma_{th}\} = \Pr\{\gamma_1 \geq \gamma_{th} \; or \; ......\gamma_M \geq \gamma_{th} | \gamma_0 \leq \gamma_{th}\} = \Pr\{\bigcup_{k=1}^{M} \gamma_k \geq \gamma_{th} | \gamma_0 \leq \gamma_{th}\} = 1 - \Pr\{\bigcap_{k=1}^{M} \gamma_k \leq \gamma_{th}\}.$$

Therefore we get

$$\Pr\{\gamma \geq \gamma_{th} | \gamma_0 \leq \gamma_{th}\} = 1 - \prod_{m=1}^{M} P_m(\gamma),$$

where $P_m(\gamma)$ is given by (15). Fig. 6 depicts this probability, using the same parameters listed in section 3. It can be seen that by using just one relay (nearest RMS to DMS) the probability of restoring the communication is 93.5%, while this probability is 99.1%. and 99.9%, when two and three relays are employed between the source and the DMS respectively.

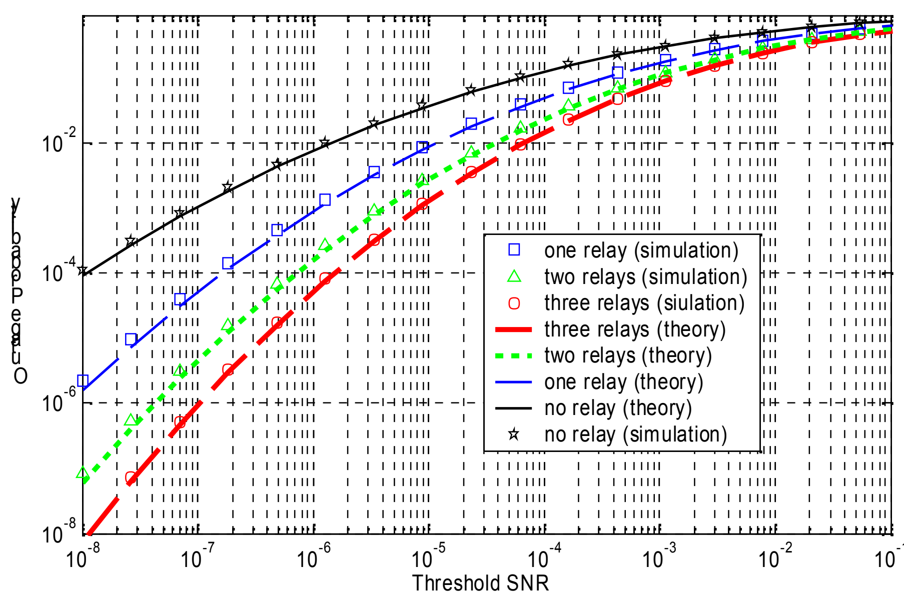

Fig. 5. Outage probability versus threshold SNR, when the relays' noise is ignored

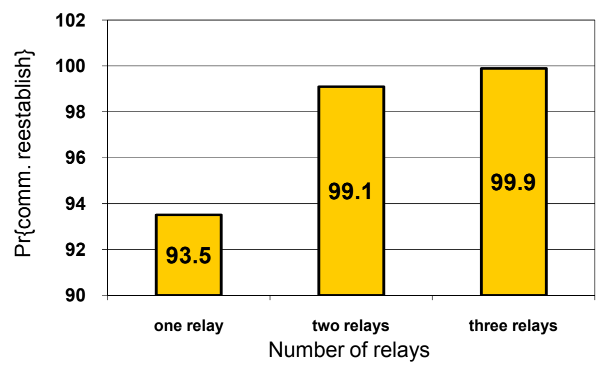

Fig. 6. Probability (%) of re-establishing communication between the source and the DMS, versus the number of relays between them

4. CONCLUSIONS

In this paper the outage probability of a proposed multi-user spatial diversity system for a wireless network was studied. The system employs one or more mobiles stations (MS) to relay the signal between the source and the destination mobile station (DMS), in a shadow-fading environment. A formula was derived for the outage probability of the received signal-to-noise ratio at DMS, when there are *M* intermediate MSs relay the signal between source and DMS. The study was accomplished by finding the probability distribution of the SNR at the DMS, when there are AGCs at the relays. It was shown that the outage probability reduces as the number of the relays increases. Finally, the probability of restoring communication between the source and the DMS as a function of number of relays between them was studied. For future work, we will examine the cases that the received signal at the relay will be regenerated before re-transmission. Also, various channels other than shadow-fade will be studied.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX

Average Distance to Destination Mobile

In this appendix the average distance of the *m* nearest MS to the DMS is determined, from the source and the DMS. Let *M* mobile stations that are potential relays (RMSs) be distributed uniformly randomly in a unit circle centered at the destination mobile station (DMS). Denote r_m to be the distance between the DMS and the mth nearest RMS to it, and let d_m be the distance between this RMS and the source (
Fig. 3). r_i are then ordered as $r_1 < r_2 < .... < r_m < ... < r_M$. The average distances $\overline{r}_m = E[r_m]$ and $\overline{d}_m = E[d_m]$ for the mth RMS are derived.

Let's begin by determining the probability density function (PDF) of r_m, $f_{r_m}(r)$. We have $f_{r_m}(r)\Delta r = \Pr\{r - \Delta r \le r_m \le r\}$ =Pr{*m*-1 RMSs are located inside the disk [0,*r-Δr*]*,* one RMS is located inside the annular [*r-Δr*, r], *M-m* RMSs are located inside the annular [r,1]}.
Knowing that $\Pr\{a \le r_i \le b\} = b^2 - a^2$, it follows that

$$f_{r_m}(r)\Delta r = m\binom{M}{m}(r - \Delta r)^{2(m-1)}(r^2 - (r - \Delta r)^2)(1 - r^2)^{M-m}.$$

Simplifying this equation we get[3]

$$f_{r_m}(r) = 2m\binom{M}{m}r^{2m-1}(1 - r^2)^{M-m}. \tag{19}$$

The expected value of r_m is then given by

$$\overline{r}_m = \int_0^1 r\, f_{r_m}(r)dr = \frac{\Gamma(M+1)\Gamma(m+\frac{1}{2})}{\Gamma(M+\frac{3}{2})\Gamma(m)} \tag{20}$$

where $\Gamma(.)$ is the gamma function. When the neighborhood cell has a radius *R*, the average distance of *m*th nearest RMS to the DMS scales to be

$$\overline{r}_m = R\frac{\Gamma(M+1)\Gamma(m+\frac{1}{2})}{\Gamma(M+\frac{3}{2})\Gamma(m)}. \tag{21}$$

Let's now calculate the expected value of the distance between the source and RMS$_m$ (

[3] Alternative proof can be found in 11, pp 147-150.

Fig. 3) We have $d_m^2 = L^2 + r_m^2 - 2r_m L\cos(\theta)$, where *L* is the distance between source and the DMS. θ is uniformly distributed over $[-\pi, \pi]$ and independent of r_m . Also we have

$$d_m = L\sqrt{1+\frac{r_m^2 - 2r_m L\cos(\theta)}{L^2}} \cong L\left(1+\frac{r_m^2 - 2r_m L\cos(\theta)}{2L^2}\right)$$

We calculate $E[r_m^2]$ by using (19), which yields $E[r_m^2] = \frac{m}{M+1}R^2$. Knowing that *E*[$\theta$]=0, the expected value of d_m is given by

$$\bar{d}_m = L\left(1+\frac{m}{2(M+1)}\frac{R^2}{L^2}\right). \tag{22}$$

Table 2 presents values for $\bar{r}_m$ and $\bar{d}_m$, when *L*=500m, *R*=250m and *M*=9, for *m*=1,2,3,4.

Table 2. Average distance of the *m*th nearest MS to the DMS, from the source and the DMS

m	**1**	**2**	**3**	**4**
$\bar{r}_m$	71 m	106 m	133 m	155 m
$\bar{d}_m$	506.3 m	512.5 m	518.8 m	525.0 m

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