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Identification of Robust Garch Models with Symmetric and Asymmetric Process with the use of Beta Volatility Coefficient and Model Accuracy Measure

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

The goal of this study was to identify a reliable GARCH model for modeling and forecasting each economic variable in Nigeria, including the price of crude oil, the consumer price index, the exchange rate, and the inflation rate. Monthly secondary data and simulated data sets were the data sets that were used. Between January 2004 and December 2020, the secondary data are covered. Beta Volatility Coefficient (BVC) model was proposed for detecting volatility in research data. Using a proposed method called Beta Volatility Coefficient (BVC) and Model Accuracy Measure (MAM) for the different sample sizes: 50, 100, 150, and 200, robust models for each variable were found. Leverage impact was there, according to the Asymmetric

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models' results. All the variables have a statistically significant value for the value. Inflation rate series is 11% more volatile than the Crude Oil Price and Exchange rate series, and when the sample size is large, the Consumer Price Index is 55% more volatile than the Crude Oil Price and Exchange rate, according to the results of the BVC of the Symmetric and Asymmetric models at the various sample sizes (200). The asymmetric "E-GARCH (1, 1) Model," the symmetric "GARCH-M (1, 1) Model," the symmetric "GARCH (1, 1) Model," and the symmetric "E-GARCH (1, 1) Model" are the identified robust models for the prediction of the Crude Oil Price series, the Inflation Rate series, the Exchange Rate series, and the Consumer Price Index series, respectively. In general, the Asymmetric GARCH model outperformed the Symmetric GARCH model for Exchange rate and Consumer Price Index, which is an improvement over earlier research. The Symmetric GARCH model outperformed the Asymmetric GARCH model for Crude Oil Price and Inflation Rate. For each variable, the found reliable models were utilized to create predictions between January 2022 and December 2024. The expected ranges for the price of crude oil are $31.82 ±1.08, the inflation rate is N14.65 ±0.03, the exchange rate is N/$756.76 ±53.84, and the consumer price index is N2.26 ±0.11.

Keywords: Univariate GARCH (M-GARCH) models; Information Criteria; Symmetric and Asymmetric process; Univariate Economic Variables; Leverage Effect; Beta Volatility Coefficient (BVC) and Model Accuracy Measure (MAM). ±.

1 Introduction

A higher level of volatility suggests that a wider range of values may be possible for the total return series. This implies that the price of the entire return series can fluctuate sharply in either direction over a little period of time. A lower level of volatility indicates a generally less erratic and more stable return series.

Ijomah and Enewari [1] claim that the variables under study lack an appropriate model formulation known as the volatility model of macroeconomic variables in the financial market. Volatility is an important component to take into account when evaluating investment options for portfolio pricing, building, hedging, and risk management even if it has been said that volatility is not the same as risk, especially when it is perceived as a certainty.

On the other hand, it can be difficult to model changes in the Crude Oil Price (COP), Exchange Rate (EXCH), Inflation Rate (INF), and Consumer Price Index (CPI). The demand for goods and services may go down or up as a result of fluctuating crude oil prices Areerat and Shoichi [2].

Volatility can be defined as the rate of variation in movement between the four variables discussed above in this study. The risk of other factors increases with the degree of exchange rate volatility on the global market, and rising crude oil prices result in higher inflation rates across the board [3].

There has been a lot of discussion about the GARCH model's various features' excessive volatility in global markets such as in the work by Engle and Kroner [4], Yi et al. [5], Daniel and Fola [6]. These four variables oscillate between two different levels. It would not be reasonable to anticipate that a Univariate GARCH model would adequately represent these unique patterns for such data. Applying linear time series models to analyze the dynamic performance of economic variables is quite prevalent [7]. The technique used is an Autoregressive (AR) model of order p, AR (p). A combination of an Autoregressive (AR) and a Moving Average (MA) model of order q, MA (q), is represented by ARMA or ARIMA, SARIMA ARCH, GARCH models, etc.

Time series analysis offers excellent chances for series detection, characterization, and modeling. The essential phase in time series is comprehending, planning, and making decisions. In order to predict the future, it is crucial to examine the sequential nature of the four series, making the Robust model the ideal option.

Several research have made substantial use of certain volatility models. Conditional Heteroscedastic (ARCH) model by Engle [8], Conditional Heteroscedastic (GARCH) model by Bollerslev [9], Conditional Heteroscedastic (GARCH-M) model by Engle and others in 1987, (E-GARCH) model by Nelson [10], Conditional Heteroscedastic (P-GARCH) model by Ding and Engle in 1993, and Conditional Heteroscedastic by Zakoian [11].

Elena and Shen [12] examined asymmetric (GARCH) models in the Threshold GARCH (T-GARCH) family and proposed the Spline T-GARCH model, which in both ARCH and GARCH terms captures high-frequency return volatility, low-frequency macroeconomic volatility, and an asymmetric reaction to prior negative news. The most well-known symmetric and asymmetric GARCH models are included in the series of parametric (GARCH) models proposed by Hentschel [7], who also highlights the connection between the models and their asymmetry treatment. Maryam and Ramanathan[13] estimated the volatility of the Malaysian stock market using empirical methodology. In reality, Symmetric and Asymmetric GARCH models are included in this classification, according to Wiri and Sibeate [14]. The symmetric model merely accounts for volatility and the risks that go along with it, but the asymmetric model accounts for hazards related to leverage and ensures that the estimates of returns on macroeconomic data are not negative [15].

The literature on volatility has long been dominated by the ARCH model. It is referred to as the (GARCH) model and was independently created by Bollerslev [16]. The exchange rate volatility between the Naira and the US dollar is examined by Ojo [17] Musa et al. [18] using GARCH. Osabuohien and Edokpa [19] use the GARCH model to analyze inflation rate volatility. Francq and, Zakoian [20] used the GARCH model to study exchange rate volatility, while Cyprian et al. [21] looked at the GARCH model of USD/KES exchange rate return.

Osabuohien and Edokpa [19] evaluated the monthly inflation rate volatility in Nigeria from 2012 to 2013 using the (GARCH) model (2013). Mathieu and Anissa [22] used the Multivariate GARCH approach to study the Frontier markets. Minovic (2008) evaluated the theoretical and empirical work for multivariate volatility processes diagnostic checking. The Ljung-Box statistics (Q-stat) of standardized residuals, those of its squared, as well as those of the cross product of standardized residuals were employed in the study by Minovic (2017) to assess the suitability of the model. Sheu and Cheng (2011) compared the effects of volatility for the China and United States (US) stock markets, respectively, on Taiwan and Hong Kong, using both the VAR and the Multivariate GARCH model for two sets of periods 1996 to 2005 and 2006 to 2009.

In order to evaluate the interconnectedness and dynamics of volatility in the corn, wheat, and soybean markets in the United States on a daily, weekly, and monthly basis encompassing 1998 to 2012, Gardebroek et al. [23] used a Multivariate GARCH technique. The Univariate GARCH models were contrasted by Efimova and Serletis [24] when they looked at the actual characteristics of the volatility of the prices of natural gas, oil, and electricity. Omorogbe and Ucheoma [25] employed GARCH models to analyze the volatility of bank stock weekly returns for six banks.

In order to choose the optimum GARCH model for each variable, this study proposed the beta volatility coefficient to assess the degree of volatility in the data sets at different sample sizes and model accuracy metrics. The following goals are looked at in order to achieve the aforementioned goal:

Employing the beta volatility coefficient (BVC) to gauge the level of volatility (or varying degrees of volatility) in the data sets, to examine the performance of the estimated symmetric and asymmetric models at various sample sizes.

The best (Robust) model for each variable should be chosen by comparing the estimated symmetric and asymmetric models at different sample sizes using Model Accuracy Measures (MAM). to forecast for January 2022 to December 2024 using the recognized reliable models.

2 Materials and Methods

The following subheadings compose the research methodology for this study: Original series data transformation, beta volatility coefficient, symmetrical and asymmetrical models Utilizing model selection criteria and Model Accuracy Measures, robust GARCH model identification using univariate GARCH models is performed (MAM).

2.1 Data Transformation

The monthly data for the Nigerian Naira/US Dollar exchange rate, inflation rate, crude oil price, consumer price index, and other variables are utilized from January 2004 to December 2020. This results in 204 data in total, which are changed in order to fit the model to logarithm returns.

Let the series $\left(X_t\right)$denoted series. Then, the returns series Y_t is

$$Y_t = \left(\frac{X_t}{X_{t-1}}\right) = \left(\frac{Series_t}{Series_{t-1}}\right) \tag{1}$$

where X_t represent the each of four series time t and X_{t-1} represent the series at time $t-1$.

2.2 Volatility Models

There are two primary categories of modeling methodologies for volatilities: symmetric and asymmetric models. In the symmetric model, the conditional variance solely depends on the size of the asset and not its sign, whereas in the asymmetric model, the impacts of a negative and positive size shock on the future volatility are different [26].

2.2.1 The volatility jumps models

The first jump volatility model was proposed by Harvey and Chakravarty (2008) and was created by rewriting GARCH (1,1) as

$$\sigma_t^2 = w + \alpha_1 z_{t-1}^2 \sigma_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{2}$$

$$\sigma_t^2 = w + \alpha_1 (z_{t-1}^2 - 1)\sigma_{t-1}^2 + (\alpha_1 + \beta_1)\sigma_{t-1}^2 \tag{3}$$

Which is finally written as:

$$\sigma_t^2 = w + \alpha_1 u_{t-1} \sigma_{t-1}^2 + \varphi_1 \sigma_{t-1}^2 \tag{4}$$

Where$\varphi_1 = \alpha_1 + \beta_1 u_t = z_{t-1}^2 - 1$ and is proportional to the score of the conditional distribution of ε_1 concerns σ_{t-1}^2. This is the Beta-GARCH model because $\frac{(u_t+1)}{(v+1)}$ has a Beta distribution and the innovations u_t are given as:

$$u_t = z_t^2 - 1 \text{ for Normal distribution, } u_t \approx N(0,1) \tag{5}$$

$$u_t = \frac{(v+1)z_t^2}{v-2+z_t^2} - 1 \text{ for Student-t distribution, } z_t \approx T(0,1,v) \tag{6}$$

$u_t = 0.5v|z_t|^v/\lambda_v^v - 1$ for Generalized Error Distribution (GED), $z_t \approx GED(0,1,v)$ and $u_t = \frac{(v+1)z_t z_t^*}{(v-2)g_t \xi^{1}} - 1$ for Skewed Student-t distribution, $z_t \approx skT(0,1,\xi,v)$

$$where\, z_t^* = sz_t + m, I_t = sign\,(z_t^*) = I(z_t^* \geq 0) - (z_t^* < 0), g_t = 1 + \frac{z_t^{*2}}{(v-2)\xi^{2I_t}},$$

$$m = \frac{\Gamma\left(\frac{v-1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\Gamma\left(\frac{v}{2}\right)}\left(\xi - \frac{1}{\xi}\right) \text{ and } s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}$$

2.2.2 Methods of Estimation of GARCH Model: Maximum Likelihood Function (MLF)

The maximum likelihood estimator is the technique used to estimate the GARCH model. The GARCH model is estimated in the following phase.

(i) Specify the mean and variance equation, example (AR(1) and GARCH(l,1) model)

$$y_t = \mu + \theta y_{t-1} + \mu_t \quad \mu \sim (0, \sigma_t^2) \tag{7}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1\mu_{t-1}^2 + \beta_1\sigma_{t-1}^2 \tag{8}$$

(ii) Estimate the likelihood function to maximize the normality assumption of disturbance terms.

$$\log L = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log(\sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T}\frac{\mu_t^2}{\sigma_t^2} \tag{9}$$

2.3 Model Selection Criteria

- The three most frequently employed information criteria are the Akaike Information Criterion (AIC), Hannan-Quinn Information Criterion (HQIC), and Schwarz Information Criterion (SIC). Following is a formula for the information criteria:

    - $\mathrm{AIC} = \ln|\Sigma_r| + \frac{2}{T}MK^2$ (10)

    - $\mathrm{HQIC} = \ln|\Sigma_r| + \frac{2\ln T}{T}MK^2$ (11)

    - $\mathrm{SIC} = \ln|\Sigma_r| + \frac{\ln T}{T}MK^2$ (12)

T is the number of observations (after accounting for lags)

M is the number of parameters estimated in each equation of the unrestricted system, including the constant. $\ln|\Sigma_r|$ is the natural log of the determinant of the covariance matrix of residuals of the restricted system. MK^2 is the number of M-Garch parameters in a model with order M

2.4 Volatility

A statistical measure of volatility is the range of returns for a certain investment, return price, or market index. Volatility in the financial markets is frequently characterized by significant swings in either direction. The volatility of an asset plays a significant role in option contract price. A statistical measure of an asset's return dispersion, volatility measures how much an asset's prices vary from the mean price. It can be measured in a number of methods, such as using beta coefficients, option pricing models, and return standard deviations. Because the price is anticipated to be less predictable, volatile assets are sometimes thought of as riskier than less volatile ones. It is a crucial factor when figuring out option prices.

2.5 Beta Volatility Coefficient (BVC)

The beta (β) of a specific stock (or return series) is one indicator of its relative market volatility. In comparison to the returns of individual return series, a beta represents the overall volatility of the entire return series. A return series' systematic risk (or volatility) as contrasted to the market as a whole is measured by the beta volatility coefficient. It is incorporated into the capital asset pricing model (CAPM), which explains how systematic risk and expected return on assets relate to one another. The CAPM approach is frequently used to value hazardous securities and to estimate asset returns while taking both the risk of the asset and the cost of capital into account. The Capital Asset Pricing Model uses it primarily (CAPM). It is a way to compare a securities or portfolio's volatility, or systematic risk, to the market as a whole.

An investor can only roughly predict how much risk a stock will add to a (supposedly) diversified portfolio using the beta data for that stock (return series).

The stock must be tied to the benchmark used in the computation for the beta Volatility Coefficient to have any value. It can gauge a stock's volatility in relation to the market's overall systemic risk. The slope of a line through a regression of data points is known statistically as beta. Each of these financial data points compares the returns of a certain stock to those of the overall market.

The activity of a security's returns as they change in response to market fluctuations is adequately described by beta. The beta of a security is derived by multiplying the covariance between the returns of the security and the market over a certain period by the market's variance.

The calculation for Beta Volatility Coefficient (BVC) is as follows:

$$\textbf{BVC}\ (\beta) = \frac{Covariance(Y_e,\ Y_m)}{Variance\ (Y_m)} \tag{13}$$

Where,

Ye = the return on an individual series (return series)
Ym = the return on the overall market (overall return series)
Covariance=how changes in a stock’s returns are related to changes in the market’sreturns (overall return series).
Variance=how far the market’s data points spreadout from their average value

2.5.1 Requirements for Beta Volatility Coefficient

Beta Volatility Coefficient (BVC) on the symmetric and asymmetric models criteria,
Beta Volatility Coefficient Equal to 1.0: indicates that its price activity is strongly correlated with other prices or it has systematic risk.
Beta Volatility Coefficient Less Than 1.0: means that the price is theoretically less volatile than other prices or tend to move more slowly than the overall prices averages.
Beta Volatility Coefficient Greater Than 1.0: This indicates that the price is theoretically more volatile than the other prices or tend to have higher volatile than the overall prices or benchmark.
Beta Volatility Coefficient is Negative: means that the price is inversely correlated to the overall prices or benchmark. This prices could be thought of as an opposite, mirror image of the benchmark’s trends, which are designed to have negative volatilities.

2.5.2 Beta Volatility Coefficient in Theory vs. Beta Volatility Coefficient in Practice

According to the beta volatility coefficient theory, the return series (or stock returns) should have a statistically normal distribution. However, there could be a big surprise in the financial markets. Returns aren't always normally distributed in reality. Consequently, what a stock's beta may suggest about its potential future movement isn't necessarily accurate.
Even though a stock's price swings may be less pronounced, one with a very low beta may nonetheless be experiencing a long-term downturn. Therefore, adding a stock in a downtrend with a low beta only reduces risk in a portfolio if the investor narrowly defines risk in terms of volatility (rather than as the potential for losses). Practically speaking, a downtrending low beta stock is unlikely to boost the performance of a portfolio.

Similar to this, a portfolio's risk will increase if a high beta stock is highly volatile in an upward trend, but it may also contribute gains. Before assuming a stock would increase or decrease risk in a portfolio, it is advised that investors who use the beta volatility coefficient to evaluate a stock also evaluate it from other viewpoints, such as fundamental or technical aspects.

For each economic indicator at different sample sizes of 50, 100, 150, and 200, the beta volatility coefficient (BVC) is utilized in this study to assess the degree of volatility of the return series fitted by the identified symmetric and asymmetric model.

2.6 Models Accuracy Measures (MAM)

To get the best fit, the model with the fewest accuracy measurements will be used (MAPD, MAD and MSD). The model that reduces the criterion is the best. Mean Absolute Percentage Deviation (MAPD), Mean Absolute Deviation (MAD), and Mean Squared Deviation are just a few of the latest selection criteria that have been offered (MSD). The study's model accuracy measurements are:

2.6.1 Mean Absolute Percentage Deviation (MAPD)

This metric assesses the correctness of fitted time series values and expresses accuracy as a percentage of the error since it is expressed as a percentage.

$$MAPD = \frac{\Sigma\left|(\mathrm{Y}_t - \hat{\mathrm{Y}}_t) / \mathrm{Y}_t\right|}{n} \mathrm{X}100 \; (\mathrm{Y}_t \neq 0) \tag{14}$$

where $\hat{\mathrm{Y}}_t$ is the fitted values of the identified symmetric and asymmetric model, Y_t is the actual series values at time t and n is the number of observations.

2.6.2 Mean Absolute Deviation (MAD)

This metric assesses the precision of time series values that have been fitted. It expresses accuracy in the same as the data. This helps to theorize the error also.

$$MAD = \frac{\sum_{t=1}^{n}\left|(\mathrm{Y}_t - \hat{\mathrm{Y}}_t)\right|}{n} \tag{15}$$

where $\hat{\mathrm{Y}}_t$, Y_t and n as defined in Equation (3.12)

2.6.3 Mean Squared Deviation (MSD)

Regardless of the model, this is calculated using the same denominator, n. This enables model comparison of MSD values. Therefore, MSD is more vulnerable than MAD to the largest forecast error.

$$MSD = \frac{\sum_{t=1}^{n}(\mathrm{Y}_t - \hat{\mathrm{Y}}_t)^2}{n} \tag{16}$$

Where, $\hat{\mathrm{Y}}_t$, Y_t and n as defined in Equation (3.12)

3 Result and Discussions

3.1 Comparisons of Estimated Univariate GARCH Models for Symmetric and Asymmetric models using Information Criteria

Symmetric:

1) ARCH (1) or GARCH (0,1) Model; If p=0 and q=1, we have

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 \tag{17}$$

where α_0 is the constant coefficient and α_1 is the coefficient of the error of conditional variance of the series.

2) GARCH (1,1) Model; If p=1 and q=1, we have

$$\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{18}$$

where α_0 and α_1 are defined in Equation (17). β_1 is known as the conditional variance since it is a one period ahead estimate for the variance.

3) GARCH-M (1,1) Model

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{19}$$

where $u_{t-1}^2 = \sigma_t e_t$ and $e_t \sim \mathrm{N}(0, \sigma_t^2)$ is the normal distribution error of the conditional variance of the series. α_0 , α_1 and β_1 are defined in Equation (18).

Note that: ARCH (1) model has two parameters estimated, while GARCH (1,1) model and GARCH-M (1,1) model have three parameters estimated.

Asymmetric models:

(1) APARCH (1,1) model: Similarly, If p=1 and q=1, we have

$$h_t = \alpha_0 + \alpha_1 \left(|e_{t-1}| - \gamma\, e_{t-1} \right) + \beta_1 h_{t-1} \tag{20}$$

and $h_t = \sigma_t^{\phi}$, where ϕ is the power transformation of the variance. It implies that

$$\begin{aligned} \sigma_t^{\phi} &= \alpha_0 + \alpha_1 \left(|e_{t-1}| - \gamma\, e_{t-1} \right) + \beta_1 \sigma_{t-1}^{\phi} \\ \sigma_t^{\phi} &= \alpha_0 + \alpha_1 |e_{t-1}| + (-\alpha_1 \gamma) e_{t-1} + \beta_1 \sigma_{t-1}^{\phi} \\ \sigma_t^{\phi} &= \alpha_0 + \alpha_1 |e_{t-1}| + \alpha_2\, e_{t-1} + \beta_1 \sigma_{t-1}^{\phi} \end{aligned} \tag{21}$$

where $\alpha_2 = -\alpha_1 \gamma$ and γ is leverage effect coefficient.

Note that: APARCH (1, 1) model has five parameters estimated α_0 , α_1 , α_2 , β_1 and ϕ.

(2) T-GARCH (1,1) model: Likewise, If p=1 and q=1, we have

$$\begin{aligned} \sigma_t^2 &= \alpha_0 + \alpha_1\, e_{t-1}^2 + \gamma\, e_{t-1}^2 e_{t-1} + \beta_1 \sigma_{t-1}^2 \\ \sigma_t^2 &= \alpha_0 + \alpha_1\, e_{t-1}^2 + \alpha_2\, e_{t-1}^2 e_{t-1} + \beta_1 \sigma_{t-1}^2 \end{aligned} \tag{22}$$

where $h_t = \sigma_t^2$; $\alpha_2 = \gamma$, γ is leverage effect coefficient.

Note that: T-GARCH (1, 1) model has four parameters estimated α_0 , α_1 , α_2 , and β_1 .

(3) E-GARCH (1,1); If p=1 and q=1, we have

$$\ln\sigma_t^2 = \alpha_0 + \alpha_1\left(\frac{[e_{t-1}] + \gamma\, e_{t-1}}{\sigma_{t-1}}\right) + \beta_1 \ln\sigma_{t-1}^2$$

$$\ln\sigma_t^2 = \alpha_0 + \alpha_1\left(\left|e_{t-1}\right|/\sigma_{t-1}\right) + \alpha_1\gamma\left(e_{t-1}/\sigma_{t-1}\right) + \beta_1 \ln\sigma_{t-1}^2 \tag{23}$$

$$\ln\sigma_t^2 = \alpha_0 + \alpha_1\left(\left|e_{t-1}\right|/\sigma_{t-1}\right) + \alpha_2\left(e_{t-1}/\sigma_{t-1}\right) + \beta_1 \ln\sigma_{t-1}^2$$

Where,

$h_t = \sigma_t^2$; $\alpha_2 = \alpha_1\gamma$, γ is leverage effect coefficient.

Note that: E-GARCH (1, 1) model has four parameters estimated α_0, α_1, α_2, and β_1.

Table 1. Comparison of the Symmetric and Asymmetric Models Selection Criteria

Variables	Model Selection Criteria	Symmetric Models			Remark	Asymmetric Models			Remark
		ARCH (1) Model	GARCH (1,1) Model	GARCH-M (1,1) Model		APARCH (1,1) model	T-GARCH (1,1) model	E-GARCH (1,1) Model	
Crude Oil Prices	AIC	7.3454	7.3219	**7.3140**	GARCH -M (1,1) Model	7.2936	7.2833	**7.2676**	E-GARCH (1, 1) Model
	BIC	7.3781	7.3708	**7.3703**		7.3752	7.3486	**7.2239**	
	HQIC	7.3586	7.3416	**7.3404**		7.3266	7.3097	**7.2941**	
Inflation Rate	AIC	7.4869	**7.2108**	7.2153	GARCH (1,1) Model	7.3419	7.2058	**7.1923**	E-GARCH (1, 1) Model
	BIC	7.4396	**7.2598**	7.2808		7.4235	7.2710	**7.2576**	
	HQIC	7.4201	**7.2306**	7.2420		7.3749	7.2322	**7.2187**	
Exchange Rate	AIC	4.2864	4.2615	**4.2537**	GARCH -M (1,1) Model	4.2256	**4.2314**	4.2334	TS-GARCH (1,1) model)
	BIC	4.3191	4.3104	**4.3100**		4.3072	**4.2967**	4.2976	
	HQIC	4.2997	4.2813	**4.2801**		4.2586	**4.2578**	4.2582	
Consumer Price Index	AIC	7.2129	**6.7645**	6.7732	GARCH (1,1) Model	6.5867	6.6087	**6.5426**	E-GARCH (1, 1) Model
	BIC	7.2456	**6.8135**	6.8385		6.6683	6.6740	**6.6079**	
	HQIC	7.2261	**6.7843**	6.7996		6.6197	6.6351	**6.5690**	

***NOTE**: Asymmetric Models have least values model selection criteria*

Six models were estimated for all the return series; three models for Symmetric model and three models for the Asymmetric. All variables were stationary at lag 1 of which the best model was selected based on information criteria.

Crude Oil Price Series: The GARCH-M (1, 1) model was selected for the Symmetric model while the E-GARCH (1, 1) model was selected for the Asymmetric process in Table 1.

Inflation Rate Series: The GARCH (1, 1) model was selected for the Symmetric model while the E-GARCH (1, 1) model was selected for the Asymmetric process in Table 1.

Exchange Rate Series: The GARCH-M (1, 1) model was selected for the Symmetric model while the T-GARCH (1, 1) model was selected for the Asymmetric process in Table 1.

Consumer Prices Index Series: The GARCH (1, 1) model was selected for the Symmetric model while the E-GARCH (1, 1) model was selected for the Asymmetric process in Table 1.

The identified models (by substitution of the estimates coefficients) are

GARCH-M (1, 1) model: $\hat{\sigma}_t^2 = 13.7557 + 0.7163u_{t-1}^2 + 0.3614\sigma_{t-1}^2 \; where \; u_{t-1}^2 = \sigma_t e_t$

E-GARCH (1,1) model: $\ln \hat{\sigma}_t^2 = 0.7173 + 0.8410\left(|e_{t-1}| / \sigma_{t-1} \right) - 0.2762\left(e_{t-1} / \sigma_{t-1} \right) + 0.6920 \ln \sigma_{t-1}^2$

for Crude Oil Prices Series (COP).

GARCH (1, 1) model: $\hat{\sigma}_t^2 = 1.8193 + 0.2731e_{t-1}^2 + 0.7303\sigma_{t-1}^2$

E-GARCH (1,1) model $\ln \hat{\sigma}_t^2 = -0.1088 - 0.0528\left(|e_{t-1}| / \sigma_{t-1} \right) + 0.0291\left(e_{t-1} / \sigma_{t-1} \right) + 0.9960 \ln \sigma_{t-1}^2$

for Inflation Rate Series (INF).

GARCH-M (1, 1) model: $\hat{\sigma}_t^2 = 1.7479 + 0.8930u_{t-1}^2 + 0.3139\sigma_{t-1}^2 \; where \; u_{t-1}^2 = \sigma_t e_t$

T-GARCH (1, 1) model: $\hat{\sigma}_t^2 = 1.7471 + 1.2673e_{t-1}^2 - 1.3680e_{t-1}^2 e_{t-1} + 0.3149\sigma_{t-1}^2$

for Exchange Rate Series (EXCH.).

GARCH (1, 1) model: $\hat{\sigma}_t^2 = -0.0511 + 0.0928e_{t-1}^2 + 0.9028\sigma_{t-1}^2$

E-GARCH (1,1) model: $\ln \hat{\sigma}_t^2 = -0.0895 - 0.0255\left(|e_{t-1}| / \sigma_{t-1} \right) - 0.00217\left(e_{t-1} / \sigma_{t-1} \right) + 1.0184 \ln \sigma_{t-1}^2$

for Consumer Price Index Series (CPI).

Based on the findings in Table 6, it was determined that, in terms of the model selection criteria, the asymmetric models outperformed the symmetric models in all of the economic variables.

In order to find the robust model for each economic indicator and the return series volatility level (or if strongly linked) with other variables, we compare the found symmetric and asymmetric models at various sample sizes, 50, 100, 150, and 200. Model Accuracy Measures, Beta Volatility Coefficient (), and simulated data sets with various sample sizes will all be used to do this.

3.2 The Simulation Data Sets Results

Macroeconomic variables will be simulated using their mean and standard deviation "$X_t \sim \mathrm{N}\left(\mu, \sigma_t^2\right)$" from the actual data sets and then identify Robust Symmetric and Asymmetric models. Then, using equation (13), their Beta Volatility Coefficient (β). (BVC) will be calculated and compared. The best model for each identified variable will be chosen using Models Accuracy Measures (MAM), which will also be used to gauge the degree of volatility in the data sets.

The following Steps are used:

1) $e_t \sim \mathrm{N}\left(0, \sigma_t^2\right) = \mathrm{N}(0,1)$ which is the normal distribution error of the conditional variance of the series.
2) Using the discovered models from above, simulate the identified robust symmetric and asymmetric models for each economic variable at different sample sizes of 50, 100, 150, and 200.
3) At the different sample sizes of 50, 100, 150, and 200, extract each return series and the overall return series using the fitted individual symmetric and asymmetric models.

4) Using Equation (13), calculate the Beta Volatility Coefficient (β) and compare the findings of the discovered Robust Symmetric and Asymmetric models for the individual series at different sample sizes, including 50, 100, 150, and 200.
5) Calculate the model accuracy metrics specified in Equations (15–17) and contrast the findings with the determined Robust Symmetric and Asymmetric models for the specific series.

3.2.1 Beta Volatility Coefficient of the identified Symmetric and Asymmetric Models at the Different Sample Sizes

First, the overall return series at the different sample sizes of 50, 100, 150, and 200 were generated using Equation (1) in and the return series of the fitted individual symmetric and asymmetric model in Figure 1. The Beta Volatility Coefficient (β).was then calculated using Equation (13), and the results are as follows:

Table 2. Beta volatility Coefficient (β) of the identified Symmetric and Asymmetric models at the various sample sizes

Sample size	COP GARCH-M (1, 1) model	COP E-GARCH (1,1) model	INF GARCH (1, 1) model	INF E-GARCH (1,1) model	EXCHGARCH-M (1, 1) model	EXCH T-GARCH (1, 1) model	CPI GARCH (1, 1) model	CPI E-GARCH (1,1)model
50	-0.23584	0.19184	0.11893	0.21645	-0.00047	0.00185	0.47514	0.56986
100	-0.26603	0.04848	0.15132	0.18555	0.00605	0.00382	0.39264	0.91613
150	-0.28275	0.04875	0.15205	0.19206	0.01799	0.01632	0.41858	0.90816
200	-0.11497	0.01857	**1.11219**	**1.10157**	0.99938	0.99875	**1.54755**	**1.51342**

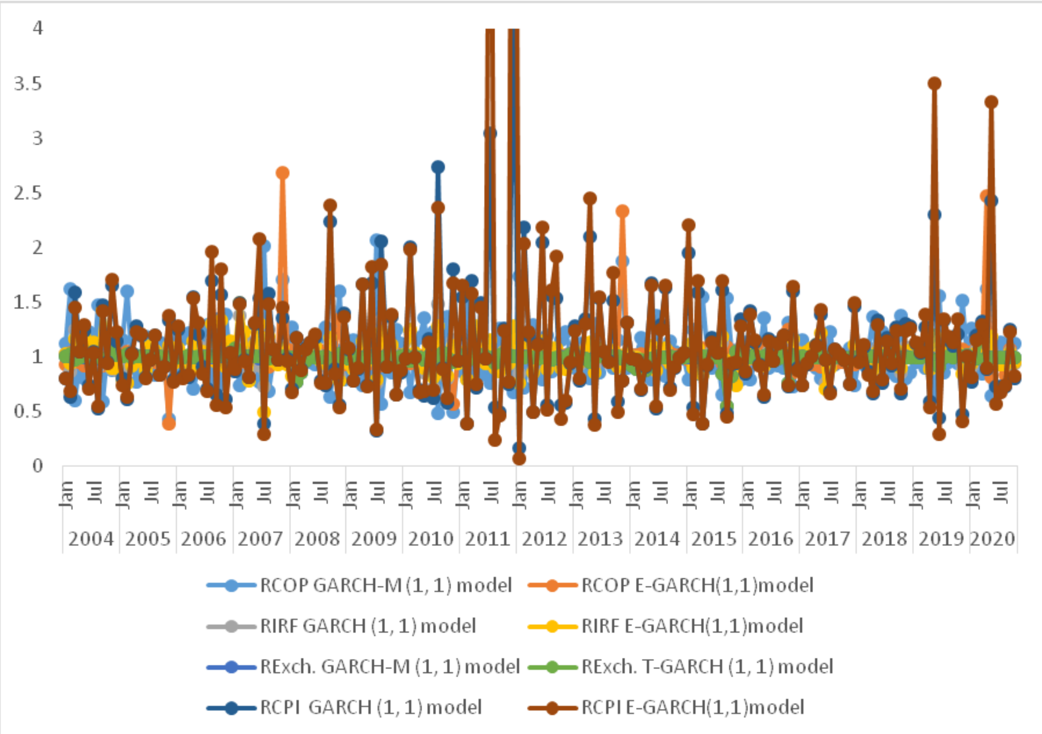

Fig. 1. The Return Series Plot of the Identified Symmetric and Asymmetric Models

When the sample size is 200, Fig. 1 and Table 2 capture the high-frequency return volatility for the identified symmetric and asymmetric models for the Consumer Prices Index Series and the Inflation Rate Series, respectively. Low-frequency return volatility when the sample sizes are 50, 100, and 150 for the Crude Oil Prices Series, the Inflation Rate Series, and the Exchange Rate Series, respectively. It displays a previous negative news in the Crude Oil Prices Series for all sample sizes using the GARCH-M (1, 1) model.

The results of the symmetric and asymmetric models for the different sample sizes are shown in Table 2. Based on the beta volatility coefficient and simulated data sets, Table 2 demonstrates that the identified models have a better fit, higher persistence of good news, and low frequency volatility component for three economic variables when the sample sizes are 50, 100, and 150 respectively.

When the sample size is 200, the beta volatility coefficients were more volatile than the two economic variables (1.11 and 1.10 for the inflation rate series, 1.54 and 1.55 for the consumer price index series). That is, the inflation rate is more volatile than two economic factors by 11% and 10%, respectively (Crude Oil Priceand Exchange rate). When n=200, the Consumer Price Index is more volatile by 54% and 55% than two economic variables (the price of crude oil and the exchange rate). The identified symmetric model [GARCH-M (1, 1) model] for Crude Oil Price has a negative Beta Volatility Coefficient, indicating an inverse correlation between the series and the three economic variables.

3.2.2 Model Accuracy Measures Comparison of the Identified Symmetric and Asymmetric Models Fitted values at the Different Sample Sizes

The section aids in determining which economic model is the most robust for each economic variable. This is accomplished by contrasting the identified values of the fitted symmetric and asymmetric models using varied sample sizes (50, 100, 150 and 200). The findings of the comparison are shown in Tables 3 to 6 below.

Table 3. Comparison of the Identified Symmetric and Asymmetric Using Models Accuracy Measures for Crude Oil Prices Series at the various sample sizes

Sample Size	COP GARCH-M (1, 1) model (Symmetric Model)			COP E-GARCH(1,1)Model (Asymmetric Model)		
	MAPD	MAD	MSD	MAPD	MAD	MSD
50	38.0608	35.4470	2095.0849	30.1821	24.3992	929.7723
100	38.0217	31.6661	1676.7841	29.3560	21.6577	701.7604
150	37.7867	30.3220	1543.4833	29.7722	21.0494	654.7549
200	36.5791	27.0042	1260.3086	29.3981	19.2091	547.7376

According to Table 3's findings, the asymmetric models outperformed the symmetric models in terms of model accuracy measures for the Crude Oil Price Series. According to this, the identified asymmetric model, or "E-GARCH(1,1) Model," is more robust than the identified symmetric model.

Table 4. Comparison of the Identified Symmetric and Asymmetric Using Models Accuracy Measures for Inflation Rate Series at the various sample sizes

Sample Size	INF GARCH (1, 1) model (Symmetric Model)			INF E-GARCH(1,1)Model (Asymmetric Model)		
	MAPD	MAD	MSD	MAPD	MAD	MSD
50	17.4759	2.6475	10.3954	12.8849	1.5962	4.1073
100	15.4280	2.1731	7.2627	11.8261	1.4542	3.3954
150	14.1850	2.0013	6.1049	10.3387	1.2929	2.8227
200	14.3974	2.1163	6.3717	9.4119	1.2230	2.5618

According to Table 4's findings, the asymmetric models outperformed the symmetric models for the inflation rate series in terms of model accuracy measures. Consequently, the identified Asymmetric model, also known as the "E-GARCH (1,1) Model"), is more reliable than the identified Symmetric model.

Table 5. Comparison of the Identified Symmetric and Asymmetric Using Models Accuracy Measures for Exchange Rate Series at the various sample sizes

Sample Size	EXCH. GARCH-M (1, 1) model (Symmetric Model)			EXCH. T-GARCH (1, 1) Model (Asymmetric Model)		
	MAPD	MAD	MSD	MAPD	MAD	MSD
50	4.2299	5.1985	44.6107	4.2539	5.2280	45.2574
100	11.2129	16.5447	429.8710	11.2537	16.6016	432.2141
150	26.8740	57.0282	14042.9640	27.0058	57.2951	14156.6805
200	64.4959	202.9326	114619.2800	64.7755	203.7842	115545.6403

According to Table 5's findings, symmetric models outperformed asymmetric models for exchange rate series in terms of model accuracy measures, indicating that the symmetric model, known as the "GARCH-M (1, 1) model," is more robust than the asymmetric model.

Table 6. Comparison of the Identified Symmetric and Asymmetric Using Models Accuracy Measures for Consumer Prices Index Series at the various sample sizes

Sample Size	CPI GARCH (1, 1) model (Symmetric Model)			CPI E-GARCH(1,1)Model (Asymmetric Model)		
	MAPD	MAD	MSD	MAPD	MAD	MSD
50	29.3365	1.3296	2.6942	30.6694	1.3393	2.7105
100	44.7025	1.2829	2.5931	45.8216	1.2855	2.5730
150	42.1453	1.1376	2.0779	43.4549	1.1405	2.0784
200	38.8254	1.0941	1.8965	39.9132	1.0871	1.9068

According to Table 6's findings, the symmetric models outperformed the asymmetric models in terms of model accuracy measures for the consumer price index series. The implication is that the identified symmetric "GARCH (1, 1) model" is more reliable than the detected asymmetric model. Additionally, the data from the fitted symmetric and asymmetric models were presented in Figs. 2 through 5 below

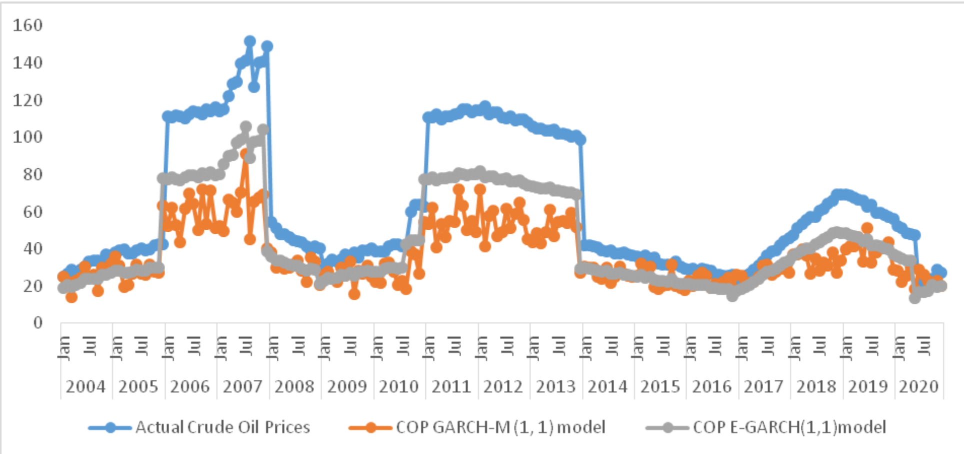

Fig. 2. Comparison Plot of the Identified Symmetric Model "GARCH-M (1, 1) Model"and Identified Asymmetric Model"E-GARCH (1,1)Model"with actual Crude Oil Price series

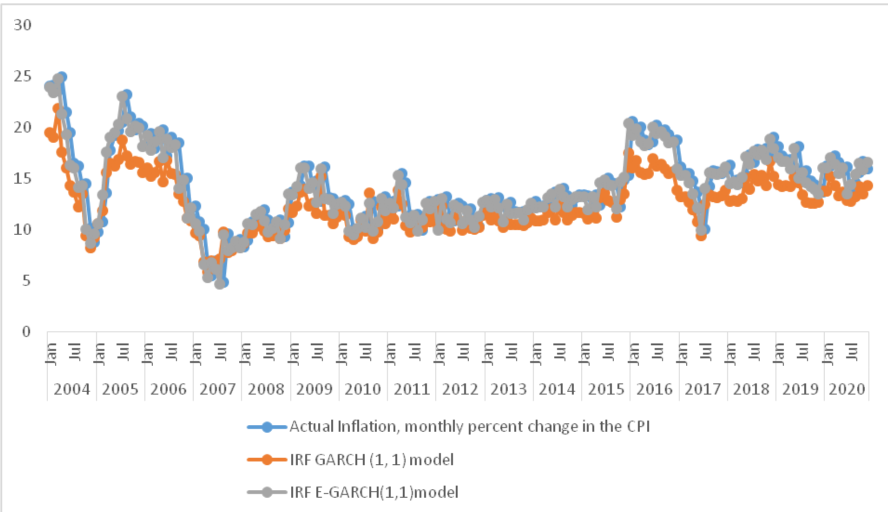

Fig. 3. Comparison Plot of the Identified Symmetric Model "GARCH (1, 1) Model" and Identified Asymmetric Model "E-GARCH (1,1)Model" with actual Inflation Rate series

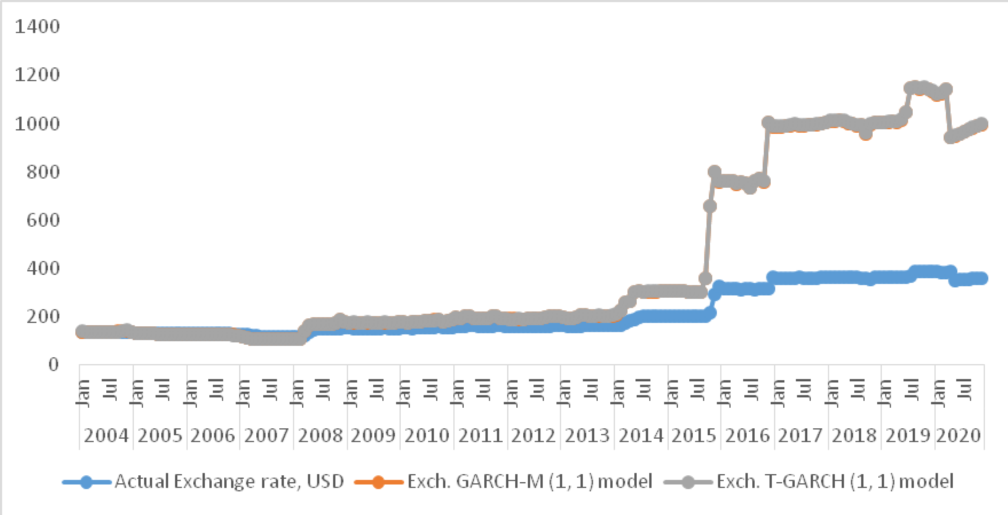

Fig. 4. Comparison Plot of the Identified Symmetric Model "GARCH-M (1, 1) Model"and Identified Asymmetric Model"T-GARCH(1,1)Model"with actual Exchange Rate series

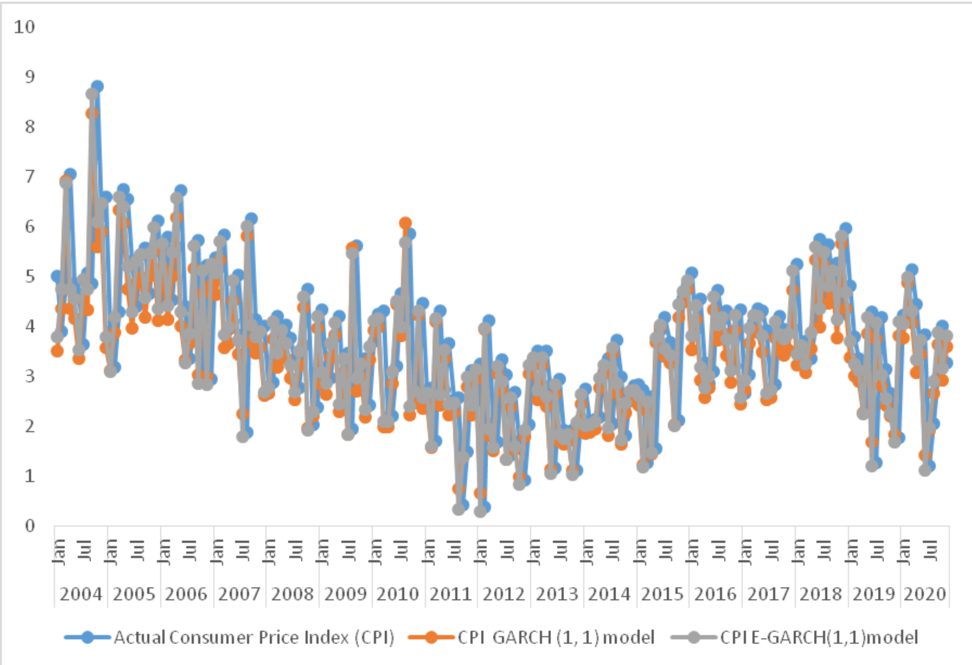

Fig. 5. Comparison Plot of the Identified Symmetric Model "GARCH (1, 1) Model"and Identified Asymmetric Model"E-GARCH(1,1)Model"with actual Consumer Price Index series

Thus, Figs. 2 to 4 confirmed that the identified asymmetric model, known as the "E-GARCH (1,1) Model," is the most reliable model for predicting changes in the price of crude oil. Other identified asymmetric models include the "GARCH-M (1, 1) model" for inflation rates, the "E-GARCH (1,1) Model" for exchange rates, and the "GARCH (1,1) Model" for changes in the price of consumer goods.

Generally speaking, the Symmetric GARCH model performs better than the Asymmetric GARCH model for the price of crude oil and the inflation rate, whereas the Asymmetric GARCH model performs better than the Symmetric GARCH model for the price of the dollar and the consumer price index

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Table 7. Forecasted Values for the identified Robust Symmetric and Asymmetric models from 2021 to 2023

			Asymmetric		Symmetric	
Year	**Month**	**Period**	**COP E-GARCH(1,1) Model**	**INF E-GARCH(1,1)Model**	**EXCH. GARCH-M (1,1) Model**	**CPI GARCH (1,1) Model**
2022	January	205	33.62553	14.60682	667.3364	2.437079
	February	206	33.52634	14.60936	672.4462	2.427333
	March	207	33.42716	14.6119	677.5561	2.41759
	April	208	33.32798	14.61444	682.6659	2.407848
	May	209	33.2288	14.61698	687.7757	2.398108
	June	210	33.12962	14.61952	692.8855	2.388369
	July	211	33.03044	14.62206	697.9953	2.378633
	August	212	32.93126	14.6246	703.1051	2.368898
	September	213	32.83209	14.62714	708.2149	2.359164
	October	214	32.73291	14.62968	713.3247	2.349433
	November	215	32.63374	14.63222	718.4345	2.339703
	December	216	32.43539	14.6373	723.5444	2.320249
2023	January	217	32.33621	14.63984	728.6542	2.310525
	February	218	32.23704	14.64238	733.764	2.300802
	March	219	32.13787	14.64492	738.8738	2.291081
	April	220	32.0387	14.64746	743.9836	2.281362
	May	221	31.93953	14.65	749.0934	2.271644
	June	222	31.84036	14.65254	754.2032	2.261928
	July	223	31.7412	14.65508	759.313	2.252214
	August	224	31.64203	14.65762	764.4229	2.242502
	September	225	31.54286	14.66016	769.5327	2.232792
	October	226	31.4437	14.6627	774.6425	2.223083
	November	227	31.34454	14.66524	779.7523	2.213376
	December	228	31.24537	14.66778	784.8621	2.203671
2024	January	229	31.14621	14.67032	789.9719	2.193967
	February	230	31.04705	14.67286	795.0817	2.184265
	March	231	30.94789	14.6754	800.1915	2.174565
	April	232	30.84873	14.67795	805.3013	2.164867
	May	233	30.74957	14.68049	810.4112	2.15517
	June	234	30.65041	14.68303	815.521	2.145475
	July	235	30.55126	14.68557	820.6308	2.135782
	August	236	30.4521	14.68811	825.7406	2.126091
	September	237	30.35295	14.69065	830.8504	2.116402
	October	238	30.2538	14.69319	835.9602	2.106714
	November	239	30.15464	14.69573	841.07	2.097028
	December	240	30.05549	14.69827	846.1798	2.087343

To fulfill the core goals of investors, stock market operators, and the government overseeing the Nigerian economic system, it is necessary to foresee the four economic factors. Future investors should pay close attention to the forecast values in Table 7, as they will help to improve economic stability in Nigeria.

Therefore, based on the expected values: Crude oil will cost between \$31.82 and \$1.08, the inflation rate will be between N\$14.65 and N\$0.03, and the consumer price index will be between N\$2.26 and N\$0.011.

4 Summary

The study looked at changes in Nigeria's exchange rate, inflation rate, CPI, and crude oil price. Monthly secondary datasets from January 2005 to December 2021 from the Central Bank of Nigeria's (CBN) statistical bulletin and simulated data sets were used in this study. The data sets were simulated using different samples of 50, 100, 150, and 200, their real data means and standard deviations, and the appropriate symmetric or asymmetric models that had been determined to be suitable. Then, for the various sample sizes of 50, 100, 150, and 200, the robust models for each variable were determined using a suggested method called Beta Volatility Coefficient (BVC) and Model Accuracy Measures (MAM). The outcome of the asymmetric models demonstrated the existence of the leverage effect, with the value being statistically significant for all variables. Inflation rate series is 11% more volatile than two other economic variables (crude oil price and exchange rate), according to the results of the BVC of the symmetric and asymmetric models at different sample sizes, while when the sample size is large, the consumer price index is 55% more volatile than the other two economic variables (crude oil price and exchange rate) (200). The asymmetric "E-GARCH(1,1) Model," the symmetric "GARCH-M (1, 1) Model," the symmetric "GARCH (1, 1) Model," and the symmetric "E-GARCH(1,1) Model" are the identified robust models for the prediction of the Crude Oil Price series, the Inflation Rate series, the Exchange Rate series, and the Consumer Price Index series, respectively. In general, the Symmetric GARCH model performs better for the Crude Oil Price and Inflation Rate than the Asymmetric GARCH model, whereas the Asymmetric GARCH model performs better for the Exchange Rate and Consumer Price Index than the Symmetric GARCH model. For each variable, these found robust models were utilized to create predictions between January 2021 and December 2023. The forecasted ranges for the price of crude oil are \$31.82 ±1.08, the inflation rate is 14.65 ±0.03, the exchange rate is N\$756.76 ±53.84, and the consumer price index is 2.26 ±0.11.

5 Conclusion

The goal of this study was to identify a reliable GARCH model for modeling and forecasting each economic variable in Nigeria, including the price of crude oil, the consumer price index, the exchange rate, and the inflation rate. Monthly secondary data and simulated data sets were the data sets that were used. Between January 2004 and December 2020, the secondary data are covered. Additionally, the Beta Volatility Coefficient (BVC), a suggested method for detecting volatility in research data, was used to model the variance and covariance stationary process and estimate the models. The series' erratic movement was found via a critical analysis of the time plot. A series is said to be stationary if its mean and variance are constant; the presence of a trend will render it non-stationary.

The lag duration of the models was calculated using the Akaike Information Criterion (AIC), Hannan-Quinn Information Criterion (HQIC), and Schwarz Information Criterion (SIC). The table demonstrates that the model based on the data is appropriate and the lag of order one (p=l) (AIC, HQIC and SIC). Although they have the minimal information criteria, the Akaike Information Criterion (AIC) and HQIC have the best lag duration of order (1).

Symmetric and asymmetric processes were used in the Univariate GARCH models for each return series. The following information criteria (AIC, SIC, and HQIC), which are described in Chapter 4, were used to compare the optimal process. The GARCH-M model and E-GARCH were the best for the series when comparing the Symmetric model to the Asymmetric process in modeling the return of Consumer Price Index. Crude Oil Price's findings indicate that the GARCH and APARCH models were the most effective for the series. The best model for the symmetric process is GARCH, while the best model for the asymmetric process is E-GARCH, when it comes to the return of inflation rate.

The series of the Exchange rate was modeled using the same GARCH and EGARCH. The stationary covariance requirement is met by all of the series. All of the ARCH models' symmetric effects had substantial volatility, which means that there was a constant upswing and downswing throughout time. The outcome demonstrated the existence of the leverage effect in all study variables that are significant at a 5% level, indicating that in the symmetric models, negative news is generally associated with higher volatility than positive news. The outcome of the asymmetric models demonstrated the existence of the leverage effect, with the value being statistically significant for all variables.

To determine the Robust Symmetric or Asymmetric models for predicting each economic indicator, the data sets were simulated at varied sampling of 50, 100, 150, and 200, utilizing their actual data sets' means and standard deviations and their recognized acceptable models. The optimal model for each variable is then determined by estimating the identified symmetric and asymmetric models' beta volatility coefficients (to gauge how volatile the data sets are).

When the sample sizes are 50, 100, and 150, the identified models have a better fit, a higher persistence of good news, and a low frequency volatility component for three economic variables, according to the results of the beta volatility coefficient of the symmetric and asymmetric models at the various sample sizes. When the sample size is higher than or equal to 200, the Beta Volatility Coefficients for the inflation rate series show that it is 11% more volatile than two other economic variables (the price of crude oil and the exchange rate), while the consumer price index is 55% more volatile.

As a result, the asymmetric "E-GARCH (1,1) model" has been recognized as the most reliable model for the prediction of Crude Oil Price series, along with the symmetric "GARCH-M (1, 1) model" for Exchange Rate series, and the symmetric "GARCH (1, 1) model" for Consumer Price Index series.

In general, the Symmetric GARCH model performs better for the Crude Oil Price and Inflation Rate than the Asymmetric GARCH model, whereas the Asymmetric GARCH model performs better for the Exchange Rate and Consumer Price Index than the Symmetric GARCH model.

Additionally, when the sample size is greater than 200, this research demonstrates that the suggested Beta Volatility Coefficient method on the identified Robust Symmetric and Asymmetric models has a better fit, persistence of good news, and higher degree of risk aversion, as well as significant effects of the economic variables used based on simulation analysis completed. For each variable, the discovered robust models were utilized to create predictions between January 2021 and December 2023. The forecasted ranges for the price of crude oil are $31.82±1.08, the inflation rate is N14.65±0.03, the exchange rate is N$756.76±53.84, and the consumer price index is N2.26±0.11.

Competing Interests

Authors have declared that no competing interests exist.

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