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Some Consequences of Bertrand's Extended Postulate

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

Bertrand's postulate establishes that for all positive integers $n > 1$ there exists a prime number between n and 2n. We consider a generalization of this theorem as: for integers $n \geq k \geq 2$ is there a prime number between kn and $(k + 1)n$? This is a generalization of Bertrand's postulate extended as proved at link 1706.01009.pdf. The example is deduced that there are at least $k - 1$ prime numbers between n and kn where n, k is a positive integers greater than 1. Then we can prove a number of hypotheses and some properties below. And here are the consequences to be deduced from it.

Keywords: Bertrand's extended postulate; prime number; integer.

1 Introduction

"In 1850, P. L. Chebyshev proved the famous Bertrand postulate (1845) that every interval [n, 2n] contains a prime (for a very elegant version of his proof, see Theorem 9.2 in" [1-5]). "Other nice proofs were given by S. Ramamujan in 1919 [6] and P. Erd″os in 1932 (reproduced in [7], pp.171-173)". "In 2006, M. El. Bachraoui [8]

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proved that every interval [2n, 3n] contains a prime, while A. Loo [9] proved the same statement for every interval [3n, 4n]". Moreover, A. Loo found a lower estimate for the number of primes in the interval [3n, 4n]. Note also that already in 1952 J. Nagura [10] proved that, "for n ≥ 25, there is always a prime between n and 65n. From his result it follows that the interval [5n, 6n] always contains a prime. In this paper we prove the following. From here we can generalize that (kn, (k+1)n) always has a prime number where n, k are positive integers greater than 1" [11-15].

1) $\left(x^2;\left(x+1\right)^2\right)$ has at least 1 prime, even 2 prime numbers.

In effect, $\left(1.1;1.2\right]$; $\left[1.2;2.2\right)$ with k equals 1.

$\left(2.2;2.3\right)$; $\left(2.3;3.3\right)$ with k equals 2.

…

$\left(x.x;x\left(x+1\right)\right)$; $\left(x\left(x+1\right);\left(x+1\right)\left(x+1\right)\right)$ with k equals x.

Thus, the Legendre conjecture is true when the other property is true.

2) Oppermann's conjecture.

+ For any integer $x>1$, there is at least one prime number between $x\left(x-1\right)$ and x^2.

In effet, $\left(1.2;2.2\right]$ with k equals 1.

$\left(2.3;3.3\right)$ with k equals 2.

…

$\left(\left(x-1\right)x;x.x\right)$ with k equals x-1.

+ For any integer $x>1$, there is at least one prime number between $x.x$ and $x\left(x+1\right)$.

In effet, $\left(2.2;2.3\right)$ with k equals 2.

$\left(3.3;3.4\right)$ with k equals 3.

…

$\left(x.x;x\left(x+1\right)\right)$ with k equals x.

Thus, the Oppermann conjecture is true when the other property is true.

3) Brocard's conjecture.

There are at least four prime numbers between P_n^2 and P_{n+1}^2, for all $n>1$, where P_n is the nth prime number.

Easy to see $P_{n+1}-P_n\geq 2$.

We consider $P_{n+1}-P_n=2$.

We must then prove that for n being a positive integer, if exists a prime number between P_n^2 and $\left(P_n+2\right)^2$.

Applying the property of element 2, we divide it into 4 intervals

$\left(P_n^2;P_n\left(P_n+1\right)\right)$; $\left(P_n\left(P_n+1\right);\left(P_n+1\right)^2\right)$; $\left(\left(P_n+1\right)^2;\left(P_n+1\right)\left(P_n+2\right)\right)$; $\left(\left(P_n+1\right)\left(P_n+2\right);\left(P_n+2\right)^2\right)$

Thus, Bertrand's conjecture is true when the other property is true.

4) $P_{n+1}-P_n<\sqrt{P_n}\Leftrightarrow P_{n+1}<\sqrt{P_n}\left(\sqrt{P_n}+1\right)$

We must then prove that for n being a positive integer, there exists a prime number between P_n and $\sqrt{P_n}\left(\sqrt{P_n}+1\right)$. The other property is true when property 2 is applied.

5) $KP_n<P_{n+\alpha}<\left(K+1\right)P_n$, It means $K<\frac{P_{n+\alpha}}{P_n}<K+1$

6) Assuming that two prime numbers p and q and have a difference of n, then there are at least 2*n* prime numbers between p^2 et q^2.

By applying the property of element 2, we divide it into 2*n* intervals.

$(+)$ $\left(P^2;P\left(P+1\right)\right)$; $\left(P\left(P+1\right);\left(P+1\right)^2\right)$; $\left(\left(P+1\right)^2;\left(P+1\right)\left(P+2\right)\right)$; $\left(\left(P+1\right)\left(P+2\right);\left(P+2\right)^2\right)$

$(+)$ $\left(\left(P+2\right)^2;\left(P+2\right)\left(P+3\right)\right)$; $\left(\left(P+2\right)\left(P+3\right);\left(P+3\right)^2\right)$; $\left(\left(P+3\right)^2;\left(P+3\right)\left(P+4\right)\right)$; $\left(\left(P+3\right)\left(P+4\right);\left(P+4\right)^2\right)$

$(+)$ $\left(\left(P+n-2\right)^2;\left(P+n-2\right)\left(P+n-1\right)\right)$; $\left(\left(P+n-2\right)\left(P+n-1\right);\left(P+n-1\right)^2\right)$; $\left(\left(P+n-1\right)^2;\left(P+n-1\right)\left(P+n\right)\right)$; $\left(\left(P+n-1\right)\left(P+n\right);\left(P+n\right)^2\right)$

Thus, property 6 is correct.

7) Andrica's conjecture

$\sqrt{P_{n+1}}-\sqrt{P_n}<1\Leftrightarrow P_{n+1}<P_n+2\sqrt{P_n}+1$

$But\ P_{n+1}<P_n+\sqrt{P_n}$ (according to the property 4)

8) Assuming that two prime numbers and have a difference of n, then there are at least mn prime numbers between p^m and q^m where m is a positive entry greater than 1.

By applying property 6 and the induction method, we obtain property 8 correctly.

9) If q is a prime number, there is less $q-1$ prime numbers between q and q^2

By applying the property of element 2, we divide it into q-1 intelvalles. $(q; 2q); (2q; 3q); \dots; ((q-1)q; q^2)$
.
So property 9 is correcte.

10) Where q is prime and m and k are natural numbers greater than 1 such that $m<k$ there is at least $(q-1)(k-m)$ prime numbers between q^m and q^k.

Applying the element property 2, we divide it into $(q-1)(k-m)$ intervalles.

$$(q^m; 2q^m); (2q^m; 3q^m); \dots; ((q-1)q^m; q^{m+1})$$
$$(q^{m+1}; 2q^{m+1}); (2q^{m+1}; 3q^{m+1}); ((q-1)q^{m+1}; q^{m+2})$$
…
$$(q^{k-1}; 2q^{k-1}); (2q^{k-1}; 3q^{k-1}); \dots; ((q-1)q^{k-1}; q^k)$$

So property 9 is correct.

11) Weak form Redmond–Sun conjecture.

With x, y, m, n having positive integers such that $x<y$ and $m<n$ there is at least am + $(y-1)(n-m)$ prime numbers between x^m and y^n with y – x = a.

By applying properties 9 and 10, we get the correct property 11.

2 Conclusions

From the fact that (n, 2n), (2n, 3n), ..., (kn, (k+1)n) in turn, there is always 1 prime number in the ranges above where n is a positive integer, we get get that (n, kn) always has at least k - 1 primes where n, k are positive integers greater than 1. For example, (n, 4n) has at least 3 primes. Besides k positive integers greater than 1, we can easily see that Andrica's conjecture is also true because k is always greater than 1.

Competing Interests

Author has declared that no competing interests exist.

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