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A Study of Distributions £ and $\mathcal{M}$ of Generalized Kenmotsu Manifold

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Authors' contributions

This is the doctoral work carried out by the author SV under the guidance of author CSB. Both authors read and approved the final manuscript.

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Abstract

The object of the present paper is to study the geometry of distributions £ and $\mathcal{M}$ of the generalized Kenmotsu manifold. Using the totally umbilicity of the distributions £ and $\mathcal{M}$, it is shown that if M is totally umbilical then it is totally geodesic. Also, the integrability of the distributions £ and M are proved. Further geometric conditions connecting the distribution $\mathcal{M}$ and submanifold M are obtained.

Keywords: Geometry of submanifold; Generalized Kenmotsu manifold; M- totally umbilical and M-totally geodesic; £-totally umbilical and £-totally geodesic; integrability of the distributions.

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1 Introduction

In 1963, Yano [1] introduced the notion of φ-structure on a C^{∞} (2n +s)-dimensional manifold $\overline{M}$ as a non-vanishing tensor field of ϕ type (1,1) on $\overline{M}$ which satisfies ϕ3 + ϕ = 0 and has constant rank r=2n. The

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almost complex (s=0) and almost contact (s=1) structures are examples of φ-structures. In 1970, S.I.Goldberg and K.Yano [2]. defined globally framed ϕ-structures, for which the subbundle kerφ is parallelizable. Then there exists a global frame { ξ_1. ....., ξ_s } for the subbundle kerφ, (the vector fields ξ_1,......,ξs are called the structure vector 1 with dual 1-forms, η_1 ,....., .., η_s such $that$

$$g(\phi X, \phi Y) = g(X, Y) - \sum_{i=1}^{s} \eta_i (X)\eta_i(Y)$$

for any vector fields X,Y in $\overline{M}$ and then the structure is called a metric φ-structure.

A wider class of globally framed φ-manifolds was introduced by [3] according to the following definition; a metric φ-structure is said to be K-structure if the fundamental 2-form Φ given by $\Phi(X,Y) = g(X, \varphi Y)$ for any vector fields X and Y on M is closed and normality condition holds , that is $[\varphi, \varphi] + 2 \sum_{i=1}^{s} d\,\eta_i \otimes \xi_i = 0,\ where\ [\varphi,\varphi]\ denotes$ the Nijenhuis tensor of Φ.

A K-manifold is called an S-manifold $if\ d\eta_\alpha = \Phi$ for α = 1,...,s. $If\ s = 1$, an S-manifold reduces to Sasakian manifold. Sasakian manifolds are manifolds of positive or zero curvature. But Kenmotsu manifolds are manifolds of negative curvature. To study manifolds with negative curvature, Bishop and O` Neill introduced the notion of warped product as a generalization of Riemannian product [3]. If M is an odd dimensional contact manifold (2n+1)the sectional curvature of plane sections containing ξ is constant say c then $if\ c > 0$ M is a homogeneous Sasakian manifold of constant holomorphic sectional curvature and if c=0, M is the Riemannian product of a line or a circle with a Kahler manifold of constant holomorphic sectional curvature. Further if $c < 0$, then the manifold is warped product space $R\times f\, C^n$. For generalized contact manifolds of dimensional (2n+s) the plane sections contain the linear span L ({ξ_i }) of the frame { ξ_1. ....., ξ_s } Further $\overline{M}$ is said to be a generalized almost Kenmotsu manifold if for all 1 ≤ i ≤ s, 1-forms η_i are closed and d $\Phi = \eta_i \wedge \Phi$. A normal generalized almost Kenmotsu manifold M is called a generalized Kenmotsu manifold.

The research work on the geometry of invariant submanifolds of contact and complex manifolds is carried out by M. Kon [4], in 1973, C. S. Bagewadi [5], in 1982, K.Yano and M. Kon [6], in 1984, and also by the authors, [7,8,9,10,11,12,13,14] etc, during [2007-2016]. Also the study of geometry of anti-invariant submanifolds is carried out by [15,16,17,18,19,20,21,22,23] in various contact manifolds. Recently the authors [24], have studied generalized Kenmotsu manifolds and also others [25,26,27,28,12,2,29,30,31] etc. Motivated by the studies of the above Authors we study the geometry of submanifolds of generalized Kenmotsu manifolds.

The paper is organised as follows: the section 2 consists of preliminaries of generalized Kenmotsu manifold, and section 3 contains the results on totally umbilicity and totally geodesicity and integrability of distributions £ and $\mathcal{M}$. Using these the geometric properties of submanifolds of generalized Kenmotsu manifolds are given.

2 Preliminaries

Let $\overline{M}$ be (2n+s)- dimensional differentiable manifold with a φ-structure of rank 2n [2]. If there exists on $\overline{M}$ vector fields ξ_i, i =,....,s and η_i s-differentiable dual 1-forms, such that

$$\phi^2 = -I + \sum_{i=1}^{s} \eta_i \otimes \xi_i, \quad \eta_i \circ \xi_j = \delta_{ij} \tag{2.1}$$

then $\overline{M}$ is c alled a φ-manifold. Moreover, we have

$$\eta_i \circ \varphi = 0, \qquad \varphi(\xi_i) = 0 \tag{2.2}$$

Further we say that $\overline{M}$ is a metric φ-manifold if there exists on $\overline{M}$ a Riemannian metric g such that

$$g(\phi X, \phi Y) = g(X, Y) - \sum_{i=1}^{s} \eta_i(X)\eta_i(Y)$$

for any X,Y ∈ T $\overline{M}$. In addition, we have

$$g(X, \phi Y) = -g(\varphi X, Y), \quad g(X, \xi_i,) = \eta_i(X) \tag{2.3}$$

A 2-form Φ defined by $\Phi(X, Y) = g(X, \phi Y)$

For any X,Y∈ T $\overline{M}$ is called the fundamental 2-form. Moreover, a metric φ-manifold is normal if $[\varphi, \varphi] + 2\sum_{i=1}^{s} d\eta_i \otimes \xi i = 0$.

A (2n+s), s ≥ 1, dimensional almost contact metric manifold $\overline{M}$ is called Kenmotsu s-manifold if it satisfies the condition [32]

$$(\overline{\nabla}_X \phi)Y = \sum_{i=1}^{s} [g(\Phi X, Y)\xi i - \Phi X \eta_i(Y)] \tag{2.4}$$

where $\overline{\nabla}$ denotes the Riemannian connection w.r.t g. on $\overline{M}$. For a generalised Kenmotsu manifold $\overline{M}$ the following formulas also hold by virtue of (2.1), (2.2) and (2.4);

$$\overline{\nabla}_X \xi = -\phi^2 X \tag{2.5}$$

$$(\overline{\nabla}_X \eta_i)Y = g(X, Y) - \sum_{i=1}^{s} \eta_i(X)\eta_i(Y)$$

for any X,Y ∈ T $\overline{M}$. We de fine the distributions £ and $\mathcal{M}$ as follows;

Definition 2.1. Denote by $\mathcal{M}$ the distribution spanned by the structure vector field ξ_i. ....., ξ_s and £ orthogonal distribution of $\mathcal{M}$ then T $\overline{M}$ = £⊕$\mathcal{M}$. If X ∈M we have ϕX = 0 and if X ∈ £ we have

$\eta_i(X) = 0$ for i = 1,.....,s i.e $\phi^2 X = -X$.

Let M be a submanifold of $\overline{M}$ Let $T_x(M)$ and $T^{\perp}{}_x(M)$ denote the tangent and normal space of M at x ∈ M respectively The Gauss and Weingarten formulas are given by

$$\overline{\nabla}_X Y = \nabla_X Y + \sigma(X, Y) \tag{2.6}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^{\perp}{}_X N \tag{2.7}$$

for any vector fields X,Y tangent to M and any vector field N normal to M , where of $\overline{\nabla}$ and ∇ are the operator of covariant differentiation on and of $\overline{M}$ and M , is $\nabla^{\perp}$ the linear connection induced in the normal space $T^{\perp}{}_{x}(M)$ Both A_N an d σ are called the Shape operator and the second fundamental form and they satisfy

$$g(\sigma(X,Y),N) = g(A_N X,Y) \tag{2.8}$$

If the second fundamental form σ of M is of the form σ(X,Y) = g(X,Y) μ, then M is called totally umbilical. where μ is the mean curvature. If the second fundamental form vanishes identically then M is said to be totally geodesic. If μ = 0, then M is said to be minimal.

3 Submanifolds of Generalized Kenmotsu Manifolds

A submanifold M of a generalized Kenmotsu manifold $\overline{M}$ is said to be invariant if the structure vector field ξ of $\overline{M}$ is tangent to M and φ($T_x(M) \subset T_x(M)$, where $T_x(M)$ is the tangent space for all x ∈ M and If φ($T_x(M) \subset T^{\perp}{}_{x}(M)$ where $T^{\perp}{}_{x}(M)$ is the no rmal space at x ∈ M then M is said to be anti-invariant in $\overline{M}$.

In this section we obtain results on totally umbilicity of £, M and invariance and anti-invariance of M. Also we study the integrability of £ and M.

Theorem 3.1. Let M be a submanifold of generalized Kenmotsu-manifold $\overline{M}$ tangent to the distribution $\mathcal{M}$. If M is $\mathcal{M}$-totally umbilical then M is $\mathcal{M}$- totally geodesic and the distribution $\mathcal{M}$ is parallel with respect to the induced connection on M.

Proof. Suppose M is tangent to M, i.e M is tangent each ξ_i. Let X,Y ∈M then φX = φY = 0.

We have from Gauss formula

$$\overline{\nabla}_X \xi_i . = \nabla_X \xi_i . + \sigma(X, \xi_i).$$

Using (2.1) (2.5) in the above we have

$$X - \sum_{i=1}^{s} \eta_i(X)\, \xi_i = \nabla_X \xi_i . + \sigma(X, \xi_i). \tag{3.1}$$

Equating tangential and normal components we have

$$X - \sum_{i=1}^{s} \eta_i(X)\, \xi_i = \nabla_X \xi_i , \qquad \sigma(X, \xi_i) = 0$$

Putting X =ξ_i, i n second equation we have $\sigma(\xi_i . \xi_i) = 0$

Let us assume that M is M-totally umbilical then

$$\sigma(X,Y) = g(X,Y)\mu$$

for any tangent vectors X,Y to $\mathcal{M}$ and μ denotes the mean curvature vector, Putting X = Y = ξ_i in the above we have

$$\sigma(\xi_i, \xi_i) = g(\xi_i, \xi_i)\mu = 0$$

This shows that $\mu = 0$ Hence $\sigma(X,Y) = g(X,Y)\mu$ implies $\sigma(X,Y) = 0$ Thus M is $\mathcal{M}$-totally geodesic.

Next from (3.1), we have $\nabla_X \xi_i = X + \sum_{i=1}^{s} \eta_i(X)\,\xi_i$

Applying φ on both sides and by virtue of (2.2) we have

$$\varphi(\nabla_X \xi_i) = -\varphi X = 0 \tag{3.2}$$

by definition of $\mathcal{M}$ Applying φ to (2.10) and by virtue of (2.1) we have

$$-\nabla_X \xi_i + \sum_{i=1}^{s} \eta_j(\nabla_X \xi_i)\xi_{j=0} \tag{3.3}$$

But

$$\eta_j(\nabla_X \xi_i) = g(\xi_j, \nabla_X \xi_i),$$

$$\nabla_X g(\xi_i, \xi_j) = 0$$

$$= g(\nabla_X \xi_i, \xi_j,) + g(\xi_i, \nabla_X \xi_j) = 0.$$

This implies $g(\xi_j, \nabla_X \xi_i) = 0$ Hence by the above (3.3) implies $(\nabla_X \xi_i) = 0$

Each ξ_i is parallel w.r.t the induced connection on M Hence the distribution $\mathcal{M}$ is parallel with respect to the induced connection on M.

Theorem 3.2. Let M be a submanifold tangent to the distribution £ of generalized Kenmotsumanifold $\overline{M}$ If M is £- totally umbilical then M is £- totally geodesic and the second fundamental form σ is parallel.

Proof; Le X,Y ∈ £, then X,Y ∈ $T\ \overline{M}$, each ξ_i *is* tangent to $\overline{M}$ *t*hen by Gauss equation

$$\overline{\nabla}_X \xi_i = \nabla_X \xi_i. + \sigma(X, \xi_i).$$

Using (2.1) and (2.5) in the above we have

$$X - \sum_{i=1}^{s} \eta_i(X)\,\xi_i = \nabla_X \xi_i. + \sigma(X, \xi_i).$$

Since X ∈ £, $\eta_i(X) = 0$ the above implies $X = \nabla_X \xi_i + \sigma(X, \xi_i)$

Equating tangential and normal components we have

$$X = \nabla_X \xi_i \quad \text{and} \quad \sigma(X, \xi_i) = 0$$

*S*ppose M is £-totally umbilical then $\sigma(X,Y) = g(X,Y)\mu$, implies, $0 = g(X,\xi_i)\mu = 0 \Rightarrow \mu = 0$

Therefore $\sigma(X,Y) = 0$. Hence M is £- totally geodesic

Further $(\overline{\nabla}_X\ \sigma)(\xi_i, \xi_i) = \nabla_X^{\perp}(\sigma(\xi_i, \xi_i) - \sigma(\nabla_X \xi_{i,,,}, \xi_i) - \sigma(\xi_i \nabla_X \xi_i) = 0 - \sigma(X, \xi_i) - \sigma(X, \xi_i) = 0$

Hence σ is parallel w.r.t. $\overline{\nabla}$ Combining Theorems 2.1 and 2.2 we can state the following Theorem.

Theorem 3.3. Let M be a submanifold of generalized Kenmotsu manifold $\overline{M}$ If M is totally Umbilical then it is totally geodesic.

Theorem 3.4. Let M be a submanifold of generalized Kenmotsu manifold $\overline{M}$ tangent to the distribution $\mathcal{M}$. Then $\mathcal{M}$ is parallel with respect the induced connection on M if and only if the submanifold is both invariant and anti-invariant submanifold of $\overline{M}$

Proof.; By hypothesis each structure vector field ξ_i i =1,……..,s is tangent to M and by Gauss

Formula we have

$$\overline{\nabla}_X \xi_i = \nabla_X \xi_i + \sigma(X, \xi_i)$$

Suppose $\mathcal{M}$ is parallel with respect to the induced connection on M then each vector field ξ_i is parallel w.r.t the induced connection on M and $\nabla_X \xi_i = 0$, i=1,.....,s

Using (2.5) and the above we have

$$X - \sum_{i=1}^{s} \eta_i(X)\, \xi_i = \nabla_X \xi_i. + \sigma(X, \xi_i). \tag{3.4}$$

Since L.H.S is tangential and R.H.S is normal; so each must be equal to zero, so $\sigma(X, \xi_i) = 0$ and $X = \sum_{i=1}^{s} \eta_i(X)\, \xi_i$ i.e X is a linear combination of the structure vector fields ξ_1..,…,, ξ_s and so $X \in M$ then $\phi X = 0$. Thus $0 \in T_x M$ as well as $0 \in T_x^{\perp} M$ because $T_x M$ and $T_x^{\perp} M$ are vector spaces. Hence M is invariant and anti-invariant submanifold of On $\overline{M}$. the other hand suppose M is invariant and anti-invariant submanifold $\overline{M}$. By definition if $X \in T_x M$ then $\phi X \in T_x M$, $\phi X \in T_x^{\perp} M$. This is possible only when $\phi X = 0$ Retracing the above steps we have

$$X - \sum_{i=1}^{s} \eta_i(X)\, \xi_i = \nabla_X \xi_i \quad \text{and} \quad \sigma(X, \xi_i) = 0$$

Applying ϕ to the first equation above we have

$$\phi X - \sum_{i=1}^{s} \eta_i(X) \phi\, \xi_i = \phi(\nabla_X \xi_i)$$

Since $\phi X = 0$ and by (2.1) we have $\phi(\nabla_X \xi_i) = 0$

Again applying ϕ to $\phi(\nabla_X \xi_i) = 0$ and by virtue of (2.1) we have $\nabla_X \xi_i = 0$ T hus ξ_i is parallel w.r.t . the induced connection Hence the distribution $\mathcal{M}$ is parallel w.r.t the induced connection.

Theorem 3.5. Let M be a submanifold of generalized Kenmotsu manifold $\overline{M}$ If $\mathcal{M}$ is normal to M then

(1) $\mathcal{M}$ is parallel w.r.t the normal connection
(2) The eigen value of the shape operator A_{ξ_i} is -1
(3) The curvature of $\mathcal{M}$ = det A_{ξ_i} is $(-1)^s$
(4) The mean curvature of M is **-s.**

Proof.; Suppose $\mathcal{M}$ is normal to M then $\xi_i \in T_x^{\perp}$M i.e each normal to M. By Weingarten formula Using (2.5) we have $X - \sum_{i=1}^{s} \eta_i (X)\phi\, \xi_i = A_{\xi_i}X + \nabla_X^{\perp}\, \xi_i$

Since M the above reduces to

$$X = A_{\xi_i}X + \nabla_X^{\perp}\, \xi_i$$

Equating tangential and normal we prove

$$\nabla_X^{\perp}\, \xi_i = 0 \tag{3.5}$$

$$A_{\xi_i}X = -\, X \tag{3.6}$$

The equation (3.5) gives the result (1) The equation (3.6) gives the result (2).

Since dimKerφ = dim M = s,

hence (3) and (4) follow from linear algebra.

Theorem 3.6. The distribution $\mathcal{M}$ is integrable

Proof;. Let X,Y ∈$\mathcal{M}$ then φX = 0, φY = 0 and X,Y ∈ of $T\,\overline{M}$

$$g([X,Y], \xi_\alpha) = g(\overline{\nabla}_X Y - \overline{\nabla}_Y X,\ \xi_\alpha) = g\big(\overline{\nabla}_X Y\ \xi_\alpha\big) - g\ (\overline{\nabla}_Y X,\ \xi_\alpha) \tag{3.7}$$

$$g([X,Y], \xi_\alpha) = Xg(Y, \xi_\alpha) - g(Y, \overline{\nabla}_X \xi_\alpha) - Y\, g(X, \xi_\alpha) + g(X, \overline{\nabla}_Y\, \xi_\alpha)$$

Since X,Y ∈$\mathcal{M}$ we have by definition of $\mathcal{M}$, $X = \sum_{i=1}^{s} \lambda_i\, \xi_i$ and $Y =, \sum_{i=1}^{s} \lambda_j\, \xi_j$

$$\text{Now} \qquad g(Y, \xi_\alpha) = g(\textstyle\sum_{i=1}^{s} \lambda_j\, \xi_j, \xi_\alpha) = \sum_{j=1}^{s} \lambda_j\, \delta_{j\alpha = }\lambda_\alpha \tag{3.8}$$

Using (3.8) and (2.5)in (3.7) we have

$$g([X,Y], \xi_\alpha) = X\, \lambda_\alpha - Y\lambda_\alpha - g(Y, X - \textstyle\sum_{i=1}^{s} \eta_{\alpha\,(X),}\, \xi_\alpha\ ) + g(X, Y - \sum_{i=1}^{s} \eta_{\alpha\,(Y),}\, \xi_\alpha) \tag{3.9}$$

$$= 0 - 0 - g(Y,X) + \eta_{\alpha\,(X)}\eta_\alpha(Y)) + g(X,Y) - \textstyle\sum_{i=1}^{s} \eta_{\alpha\,(Y),}\, \eta_\alpha(X) = 0$$

Hence [X,Y] ∈$\mathcal{M}$. According to definition of $\mathcal{M}$ we must also show φ([X,Y]) = 0;

$\varphi([X,Y]) = (\overline{\nabla}_X Y - \overline{\nabla}_Y X) = (\overline{\nabla}_X \varphi)Y + \overline{\nabla}_X \varphi Y - (\overline{\nabla}_Y\, \varphi)X - \overline{\nabla}_Y\, \varphi X$

Since φX = φY = 0. Using (2.4) in (3.9) we have

$\varphi([X,Y]) = \sum_{i=1}^{s}\ g(\Phi X, Y)\, \xi_i - \Phi X \eta_i(Y) - g(\Phi Y, X)\, \xi_i) + \Phi Y \eta_i(X = 0)$

Theorem 3.7.; The distribution £ is integrable.

Proof.; Let X,Y ∈ £ then X,Y ∈ $T\ \overline{M}$

Consider $g([X,Y], \xi_\alpha) = g(\overline{\nabla}_X Y - \overline{\nabla}_Y X, \xi_\alpha)$
$= g(\overline{\nabla}_X Y \ \xi_\alpha) - g(\overline{\nabla}_Y X, \xi_\alpha)$

$$g([X,Y], \xi_\alpha) = Xg(Y, \xi_\alpha) - g(Y, \overline{\nabla}_X \xi_\alpha) - Y g(X, \xi_\alpha) + g(X, \overline{\nabla}_Y \xi_\alpha)$$

Since X,Y ∈ £, $\phi^2 X = -X$, , $\phi^2 X = -Y$, $\eta_\alpha(X = \eta_\alpha(Y) = 0$

Using these and (2.5) in the above and simplifying we have $g([X,Y], \xi_\alpha) = 0$.

Therefore [X,Y] ∈ £. Hence £ is integrable.

4 Conclusion

This paper gives the totally umbilicity and totally geodesicity of the submanifold M of generalised Kenmotsu manifold $\overline{M}$ via the totally umbilicity and totally geodesity of the distributions $\mathcal{M}$ and £ formed by the characterstic vector fields $\xi_1 .., ..., \xi_s$ and1-forms, η_1 ,....., .., η_s dual to $\xi_1 .., ..., \xi_s$ *of g*eneralised Kenmotsu manifold $\overline{M}$. It also connects eigen values with curvature and mean curvatures of the submanifold M and distribution $\mathcal{M}$ which is justifiable .It also connects invariance and anti-invariance of the submanifold M with parallelism of the distribution $\mathcal{M}$ w.r.t the induced connection of $\overline{M}$ on M. Also it deals with integrablity of $\mathcal{M}$ and £ via the submanifold M of generelised Kenmotsu manifold $\overline{M}$.

Competing Interests

Authors have declared that no competing interests exist.

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