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Properties of *T*–Anti-Fuzzy Ideals of a ℓ–Near-Ring

G. Chandrasekaran[1*], B. Chellappa[2] and M. Jeyakumar[3]

[1]Department of Mathematics, Sethupathi Govt Arts College, Ramanathapuram – 623 502, Tamilnadu, India.
[2]Nachiappa Suvamical Arts and Science College, Karaikudi – 630 003, Tamilnadu, India.
[3]Department of Mathematics, Alagappa University Evening College, Rameswaram – 623 526, Tamilnadu, India.

Authors' contributions

This work was carried out in collaboration between all authors. Author GC designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. All authors read and approved the final manuscript.

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Abstract

In this paper, we define Anti-fuzzy ideal of a ℓ-near-ring in R and T-anti-fuzzy ideal of a ℓ-near-ring in R . we made an attempt to study the properties of T-anti-fuzzy ideal of a ℓ-near-ring, union of T-anti-fuzzy ideals of ℓ-near-ring, join of T-anti-fuzzy ideal of a ℓ-near-ring in R, join of a family of T-anti-fuzzy ideal of a ℓ-near-ring in R and family of union of T-anti-fuzzy ideal of ℓ-near-ring in R .

Keywords: Fuzzy subset; near-ring; ideal, ℓ-ring, ℓ-near-ring; T-fuzzy ideal; anti-fuzzy ideal; anti-fuzzy ideal of ℓ-near-ring; T-anti-fuzzy ideal; join of T-anti-fuzzy ideal.

1 Introduction

The concept of fuzzy sets was initiated by Zadeh LA. [1] in 1965. Wang-jin Liu. [2] has studied fuzzy ideals of a ring and many researchers are engaged in extending the concepts. Abou-Zaid S. [3] introduced the

**Corresponding author: E-mail: gurunivedhasekarani@gmail.com;*

notion of a fuzzy subnear-ring, and studied fuzzy ideals of a near-ring, and many followers discussed further properties of fuzzy ideals in near-rings. In Biswas R. [4] introduced the concept of anti-fuzzy subgroups of groups, and Kyung Ho KIM, Young Bae JUN. [5] studied the notion of anti-fuzzy *R*-subgroups of near-ring. In Kim KH, Jun YB, Yon YH. [6] introduced an anti-fuzzy ideal in Near-Rings. Triangular norms were introduced by Schweizer and Sklar [7,8] to model the distances in probabilistic metric spaces. In fuzzy sets theory, triangular norm (*t*-norm) is extensively used to model the logical connective conjunction (AND). There are many applications of triangular norms in several fields of mathematics and artificial intelligence. Dheena P, Mohanraaj G. [9] have studied several properties of T-fuzzy ideals of rings and T-fuzzy ideals of near-rings. We extended the results of Akram M. [10] to Γ-near-rings. Prakashmanimaran J, Chellappa B, Jeyakumar M. [11] introduced T-anti-fuzzy right ideals of ℓ-ring.

In this paper we define, characterize and study of T-anti-fuzzy right and left ideals. we define anti-fuzzy ideal of a ℓ-near-ring in R and T-anti-fuzzy right ideals of ℓ-near-ring in R. We discuss some of its properties. We have shown that properties of T-anti-fuzzy ideal of a ℓ-near-ring, union of T-anti-fuzzy ideals of ℓ-near-ring, join of T-anti-fuzzy ideal of a ℓ-near-ring in R, family of join of T-anti-fuzzy ideal of a ℓ-near-ring in R and family of union of T-anti-fuzzy ideal of ℓ-near-ring in R. Some of these works may be noted in [12,13,14,15,16,17,18,19,20,21,22,23,24,25,26].

We now reminding some fundamental definitions, notations and basic results that will be used throughout this paper.

Definition: 1

A non-empty set R is called a near-ring with two binary operations "+" and " . " satisfying the following axioms: $(R,+)$ is a group ; $(R,.)$ is a semigroup and $(x+y).z = x.z + y.z$, for all x, y, z in R (i.e. Multiplicative is left distributive with respect to addition) We denote $x.y$ by xy .

Definition: 2

A non-empty set R is called lattice ordered near-ring or ℓ-near-ring if it has four binary operations "+", " . ", $\vee$, $\wedge$ defined on it and satisfy the following axioms: $(R,+)$ is a group ; $(R,.)$ is a semigroup; $(R,\vee,\wedge)$ is a lattice ; and

(i) $x.(y+z) = x.y + x.z$, for all x, y, z in R

(ii) $x+(y\vee z)=(x+y)\vee(x+z)$; $x+(y\wedge z)=(x+y)\wedge(x+z)$

$(y\vee z)+x=(y+x)\vee(z+x)$; $(y\wedge z)+x=(y+x)\wedge(z+x)$

(iii) $x\cdot(y\vee z)=(xy)\vee(xz)$; $x\cdot(y\wedge z)=(xy)\wedge(xz)$

$(y\vee z)\cdot x=(yx)\vee(zx)$; $(y\wedge z)\cdot x=(yx)\wedge(zx)$, for all x, y, z in R and $x\geq 0$

Example: 1

$(nZ,+,\cdot,\vee,\wedge)$ is a ℓ-near-ring, where Z is the set of all integers and $n\in Z$

Definition: 3

A mapping from a nonempty set X to $[0,\ 1]$, $\mu : X\rightarrow[0,\ 1]$ is called a fuzzy subset of X .

Definition: 4

A fuzzy subset μ of a lattice ordered ring (or ℓ-ring) R is called an anti-fuzzy sub ℓ-ring of R if the following conditions are satisfied

(i) $\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$

(ii) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$

(iii) $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$

(iv) $\mu(x \wedge y) \leq \max\{\mu(x), \mu(y)\}$, for all x, y in R.

Example: 2

Consider the fuzzy subset μ of the ℓ-ring $(Z, +, \cdot, \vee, \wedge)$

$$\mu(x) = \begin{cases} 0.4 & if \;\; x \in \langle 3 \rangle \\ 0.8 & if \;\; x \in Z - \langle 3 \rangle \end{cases}.$$

Then, μ is an anti-fuzzy ℓ-sub ring.

Example: 3

Consider a fuzzy subset μ of the ℓ-ring $(Z, +, \cdot, \vee, \wedge)$.

$$\mu(x) = \begin{cases} 0.3 & if \;\; x \in \langle 3 \rangle \\ 0.8 & if \;\; x \in Z - \langle 3 \rangle \end{cases}.$$

Then, μ is not an anti-fuzzy ℓ-sub ring.

For example, let $x = 3$ and $y = 7$, then $x + y = 10$. Here $\mu(x) = 0.8$ and $\mu(y) = 0.8$.
Therefore, $\max\{\mu(x), \mu(x)\} = \max\{0.8, 0.8\} = 0.8$. But $\mu(x+y) = 0.3$.
Hence, $\mu(x+y) \leq \max\{\mu(x), \mu(y)\}$. Thus, μ is not an anti-fuzzy ℓ-sub ring of R.

Definition: 5

Let R be a ℓ-near-ring. A nonempty subset $(I, +)$ of $(R, +)$ is called a left ideal if $x.(y+i) - x.y \in I$ for all $x, y \in R$ and $i \in I$; a right ideal if $i.x \in I$ for all $x \in R$ and $i \in I$; an ideal, if it is both a left ideal and a right ideal of R.

Definition: 6

A fuzzy set μ in a near-ring R is said to be an anti-fuzzy ideal of R, if the following conditions are satisfied,

(i) $\mu(x-y) \leq \max(\mu(x), \mu(y))$

(ii) $\mu(y+x-y) \leq \mu(x)$

(iii) $\mu(xy) \leq \mu(y)$; $\mu(xy) \leq \mu(x)$

(iv) $\mu((x+z)y - x\,y) \leq \mu(z)$ for all $x, y, z \in R$.

Proposition: 1

If μ is an anti-fuzzy ideal of R then, $\mu(0) \leq \mu(x)$ for all $x \in R$.

Definition: 7

A fuzzy subset μ of a ℓ-near-ring R is called an anti-fuzzy ideal, if the following conditions are satisfied,

(i) $\mu(x-y) \leq \max(\mu(x), \mu(y))$

(ii) $\mu(y+x-y) \leq \mu(x)$

(iii) $\mu(xy) \leq \mu(y)$; $\mu(xy) \leq \mu(x)$

(iv) $\mu((x+z)y - x\,y) \leq \mu(z)$

(v) $\mu(x \vee y) \leq \max(\mu(x), \mu(y))$

(vi) $\mu(x \wedge y) \leq \max(\mu(x), \mu(y))$ for all $x, y, z \in R$.

Example: 4

Consider a near-ring $R = \{a, b, c, d\}$ with the following Cayley's tables:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

·	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

We define an anti-fuzzy subset $\mu : R \to [0,1]$ by $\mu(a) < \mu(b) < \mu(d) = \mu(c)$

Then, μ is an anti-fuzzy right (resp. left) ideal of R .

Definition: 8

A mapping $T : [0,\ 1] \times [0,\ 1] \to [0,\ 1]$ is called a triangular norm [t – norm], if and only if, it satisfies the following conditions:

(i). $T(x,\ 1) = T(1,\ x) = x$, for all $x \in [0,\ 1]$

(ii). $T(x,\ y) = T(y,\ x)$, for all x, $y \in [0,\ 1]$

(iii). $T(x, T(y, z)) = T(T(x, y), z)$

(iv). $T(x,\ y) \leq T(x,\ z)$, whenever $y \leq z$.

Proposition: 2

The *minimum* T-norm ($\min T$ -norm) is defined by $T(a, b) = \min\{a, b\}$.

Definition: 9

A fuzzy subset μ of a ring R is called T-anti-fuzzy right (resp. left) ideal if

(i) $\mu(x-y) \le T\ (\mu(x), \mu(y))$

(ii) $\mu(xy) \le \mu(x)$ (resp. left $\mu(xy) \le \mu(y)$), for all x, y in R .

Definition: 10

A fuzzy subset μ of a ring R is called T-anti-fuzzy ideal, if it is satisfied both right and left ideals.

Proposition: 3

Every anti-fuzzy right ideal of a ring R is an T-anti-fuzzy right ideal.

Definition: 11

A fuzzy subset μ of a ℓ-ring R is called an T-anti-fuzzy ideal, if the following conditions are satisfied,

(i) $\mu(x-y) \le T(\mu(x), \mu(y))$

(ii) $\mu(xy) \le \mu(x)$; $\mu(xy) \le \mu(y)$

(iii) $\mu(x \vee y) \le T(\mu(x), \mu(y))$

(iv) $\mu(x \wedge y) \le T(\mu(x), \mu(y))$, for all $x, y \in R$.

Definition: 12

A fuzzy subset μ of a near-ring R is called an T-anti-fuzzy ideal, if the following conditions are satisfied,

(i) $\mu(x-y) \le T(\mu(x), \mu(y))$

(ii) $\mu(y+x-y) \le \mu(x)$

(iii) $\mu(xy) \le \mu(y)$; $\mu(xy) \le \mu(x)$

(iv) $\mu((x+z)y - x\,y) \le \mu(z)$, for all $x, y, z \in R$.

Proposition: 4

Every T-fuzzy ideal of a near-ring R is a T-fuzzy sub near-ring of R .
Converse of Proposition 1 may not be true in general as seen in the following example.

Example: 5

From example: 4

Let $T:[0,1]\times[0,1] \to [0,1]$ be a function. defined by $T(x,y) = \max\{x+y-1, 0\}$ which is a t – norm, for all $x, y \in [0,1]$. By routine calculations, it is easy to check that μ is a T-anti-fuzzy sub near-ring of R . It is clear that μ is also left T-anti-fuzzy ideal of R. But μ is not T-anti-fuzzy right ideal of R, since, $\mu((c+d)d-cd)=\mu(d)<\mu(b)$.

Definition: 13

A fuzzy subset μ of a ℓ-near-ring R is called an T-anti-fuzzy ideal, if the following conditions are satisfied,

(i) $\mu(x-y)\le T(\mu(x),\mu(y))$

(ii) $\mu(y+x-y)\le\mu(x)$

(iii) $\mu(xy)\le\mu(y)$; $\mu(xy)\le\mu(x)$

(iv) $\mu((x+z)y-x\,y)\le\mu(z)$

(v) $\mu(x\vee y)\le T(\mu(x),\mu(y))$

(vi) $\mu(x\wedge y)\le T(\mu(x),\mu(y))$ for all $x,y,z\in R$.

Example: 6

Let $(R=\{a,b,c\},+,\cdot,\vee,\wedge)$ be a ℓ-near-ring.

Consider an anti-fuzzy subset μ of the ℓ-ring R

$$\mu(x)=\begin{cases}0.2 & if\ x=a\\ 0.5 & if\ x=b.\\ 0.8 & if\ x=c\end{cases}$$

Then, μ is an T-anti-fuzzy ideal of ℓ-near-ring R.

Definition: 14

Let μ and λ be the anti-fuzzy subsets of a set X. An anti-fuzzy subset $\mu\cup\lambda$ is defined as $(\mu\cup\lambda)(x)=\max\{\mu(x),\lambda(x)\}$.

Definition: 15

Let μ and λ be the anti-fuzzy subsets of a set X. An anti-fuzzy subset $\mu\vee\lambda$ is defined as $(\mu\vee\lambda)(x)=T(\mu(x),\lambda(x))$.

Example: 7

Let $R=\{0,a,b,c\}$ .We have $(\mu\vee\lambda)(x)=T(\mu(x),\lambda(x))$, for all x in R.

Define $t-$norm T by $T(p,q)=\max(p,q)$, for all p,q in $[0,1]$.

Define an anti-fuzzy subset

$\mu : R \to [0, 1]$ by $\mu(0) = 0.9$ and $\mu(a) = \mu(b) = \mu(c) = 0.4$, where $0, a, b, c \in R$.

Then $\mu = \{(0, 0.9), (a, 0.4), (b, 0.4), (c, 0.4)\}$.

Again, define an anti-fuzzy subset

$\lambda : R \to [0, 1]$ by $\lambda(0) = 0.7$, $\lambda(a) = 0.6$, $\lambda(b) = 0.5$ and $\lambda(c) = 0.4$ where $0, a, b, c \in R$.

Then $\lambda = \{(0, 0.7), (a, 0.6), (b, 0.5), (c, 0.4)\}$.

Let $0, a, b, c$ in R

Then, $(\mu \vee \lambda)(0) = T(\mu(0), \lambda(0)) = T(0.9, 0.7) = \max(0.9, 0.7) = 0.9$

Now, $(\mu \vee \lambda)(a) = T(\mu(a), \lambda(a)) = T(0.4, 0.6) = \max(0.4, 0.6) = 0.6$

$(\mu \vee \lambda)(b) = T(\mu(b), \lambda(b)) = T(0.4, 0.5) = \max(0.4, 0.5) = 0.5$

$(\mu \vee \lambda)(c) = T(\mu(c), \lambda(c)) = T(0.4, 0.4) = \max(0.4, 0.4) = 0.4$

So, $\mu \vee \lambda = \{(0, 0.9), (a, 0.6), (b, 0.5), (c, 0.4)\}$ is the anti-fuzzy subset of R.

That is, $\mu \vee \lambda : R \to [0, 1]$, defined by $(\mu \vee \lambda)(0_R) = 0.9$.

$$(\mu \vee \lambda)(x) = \begin{cases} a & \text{if } x = 0.6 \\ b & \text{if } x = 0.5 \\ c & \text{if } x = 0.4, \text{ for all } 0_R \neq x \end{cases}$$

Thus, $\mu \vee \lambda$ is an T-anti-fuzzy ideal of a ℓ-near-ring R.

Definition: 16

Let μ and λ be T-anti-fuzzy ideals of a $\ell -$ near-ring R. Then, $\mu \cup \lambda$, is an T-anti-fuzzy right ideal is defined by $(\mu \cup \lambda)(x-y) = \max(\mu(x-y), \lambda(x-y))$, for all $x, y \in R$.

Definition: 17

A fuzzy set μ of a ℓ-near-ring R has the ***supremum*** property if for any subset N of R, there exists a $a_0 \in N$ such that $\mu_A(a_0) = \sup_{a \in N} \mu_A(a)$.

Theorem: 1

Every anti-fuzzy ideal of a ℓ-near-ring R is an T-anti-fuzzy ideal in ℓ-near-ring R.

Theorem: 2

If μ and λ are T-anti-fuzzy ideals of a ℓ-near-ring R, then $\mu \vee \lambda$ is an T-anti-fuzzy ideal of a ℓ-near-

ring R.
Proof:

Given μ and λ are T-anti-fuzzy ideals of a ℓ-near-ring R,

Let $x, y, z \in R$

(i) $$(\mu \vee \lambda)(x-y) = T(\mu(x-y), \lambda(x-y))$$
$$\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y)))$$
$$= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y))$$
$$= T(T(T(\mu(x), \lambda(x)), \mu(y)), \lambda(y))$$
$$= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y)))$$
$$= T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))$$

Therefore, $(\mu \vee \lambda)(x-y) \leq T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))$, for all $x, y \in R$.

(ii) Since, $\mu(y+x-y) \leq \mu(x)$ and $\lambda(y+x-y) \leq \lambda(x)$
$$(\mu \vee \lambda)(y+x-y) = T(\mu(y+x-y), \lambda(y+x-y))$$
$$\leq T(\mu(x), \lambda(x))$$
$$= (\mu \vee \lambda)(x)$$

Therefore, $(\mu \vee \lambda)(y+x-y) \leq (\mu \vee \lambda)(x)$, for all $x, y \in R$.

(iii) Since $\mu(xy) \leq \mu(x)$ and $\lambda(xy) \leq \lambda(x)$
$$(\mu \vee \lambda)(xy) \leq T(\mu(xy), \lambda(xy)),$$
$$\leq T(\mu(x), \lambda(x)) \leq (\mu \vee \lambda)(x)$$

Therefore, $(\mu \vee \lambda)(xy) \leq (\mu \vee \lambda)(x)$, for all $x, y \in R$.

(iv) Since, $\mu((x+z)y - x\,y) \leq \mu(z)$ and $\lambda((x+z)y - x\,y) \leq \lambda(z)$
$$(\mu \vee \lambda)((x+z)y - x\,y) = T(\mu((x+z)y - x\,y), \lambda((x+z)y - x\,y))$$
$$\leq T(\mu(z), \lambda(z)) \leq (\mu \vee \lambda)(z)$$

Therefore, $(\mu \vee \lambda)((x+z)y - x\,y) \leq (\mu \vee \lambda)(z)$, for all $x, y \in R$.

(v) $$(\mu \vee \lambda)(x \vee y) = T(\mu(x \vee y), \lambda(x \vee y))$$
$$\leq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y)))$$
$$= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y))$$
$$= T(T(T(\mu(x), \lambda(x)), \mu(y)), \lambda(y))$$
$$= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y)))$$
$$= T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))$$

Therefore, $(\mu \vee \lambda)(x \vee y) \leq T((\mu \vee \lambda)(x), (\mu \vee \lambda)(y))$, for all $x, y \in R$.

(vi) $(\mu\vee\lambda)(x\wedge y) = T(\mu(x\wedge y), \lambda(x\wedge y))$

$$\le T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y)))$$
$$= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y))$$
$$= T(T(T(\mu(x), \lambda(x)), \mu(y)), \lambda(y))$$
$$= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y)))$$
$$= T((\mu\vee\lambda)(x), (\mu\vee\lambda)(y))$$

Therefore, $(\mu\vee\lambda)(x\wedge y) \le T((\mu\vee\lambda)(x), (\mu\vee\lambda)(y))$, for all $x, y\in R$.

Thus $\mu\vee\lambda$, is an T-anti-fuzzy right ideal of a ℓ-near-ring R.

Theorem: 3

If μ and λ are T-fuzzy ideals of a ℓ-near-ring R, then $\mu\cup\lambda$ is an T-anti-fuzzy ideal of a ℓ-near-ring R

Proof:

Given μ and λ are T-anti-fuzzy ideals of a ℓ-near-ring R

Let $x, y, z\in R$

(i). $(\mu\cup\lambda)(x-y) = \max\{\mu(x-y), \lambda(x-y)\}$

$$\le \max\{\max\{\mu(x), \mu(y)\}, \max\{\lambda(x), \lambda(y)\}\}$$
$$= \max\{\max\{\max\{\mu(x), \mu(y)\}, \lambda(x)\}, \lambda(y)\}$$
$$= \max\{\max\{\max\{\mu(x), \lambda(x)\}, \mu(y)\}, \lambda(y)\}$$
$$= \max\{\max\{\mu(x), \lambda(x)\}, \max\{\mu(y), \lambda(y)\}\}$$
$$= \max\{(\mu\cup\lambda)(x), (\mu\cup\lambda)(y)\}$$

Therefore, $(\mu\cup\lambda)(x-y) \le \max\{(\mu\cup\lambda)(x), (\mu\cup\lambda)(y)\}$,for all $x, y\in R$

(ii). Since, $\mu(y+x-y) \le \mu(x)$ and $\lambda(y+x-y) \le \lambda(x)$

We have, $(\mu\cup\lambda)(y+x-y) = \max(\mu(y+x-y), \lambda(y+x-y))$

$$\le \max(\mu(x), \lambda(x)) = (\mu\cup\lambda)(x)$$

Therefore, $(\mu\cup\lambda)(y+x-y) \le (\mu\cup\lambda)(x)$, for all $x, y\in R$

(iii). Since, $\mu(xy) \le \mu(x)$ and $\lambda(xy) \le \lambda(x)$

We have, $(\mu\cup\lambda)(xy) \le \max\{\mu(xy), \lambda(xy)\}$

$$\le \max\{\mu(x), \lambda(x)\}$$
$$\le (\mu\cup\lambda)(x)$$

Therefore, $(\mu\cup\lambda)(xy) \le (\mu\cup\lambda)(x)$, for all $x, y\in R$

(iv) Since, $\mu((x+z)y-x\ y) \le \mu(z)$ and $\lambda((x+z)y-x\ y) \le \lambda(z)$

$$(\mu\cup\lambda)((x+z)y-x\ y) = \max\left(\mu((x+z)y-x\ yy), \lambda((x+z)y-x\ y)\right)$$
$$\le \max(\mu(z), \lambda(z)) \le (\mu\cup\lambda)(z)$$

Therefore, $(\mu\cup\lambda)((x+z)y-x\ y) \le (\mu\cup\lambda)(z)$, for all $x, y\in R$

(v). $$(\mu\cup\lambda)(x\vee y) = \max\{\mu(x\vee y), \lambda(x\vee y)\}$$
$$\le \max\{\max\{\mu(x), \mu(y)\}, \max\{\lambda(x), \lambda(y)\}\}$$
$$= \max\{\max\{\max\{\mu(x), \mu(y)\}, \lambda(x)\}, \lambda(y)\}$$
$$= \max\{\max\{\max\{\mu(x), \lambda(x)\}, \mu(y)\}, \lambda(y)\}$$
$$= \max\{\max\{\mu(x), \lambda(x)\}, \max\{\mu(y), \lambda(y)\}\}$$
$$= \max\{(\mu\cup\lambda)(x), (\mu\cup\lambda)(y)\}$$

Therefore, $(\mu\cup\lambda)(x\vee y) \le \max\{(\mu\cup\lambda)(x), (\mu\cup\lambda)(y)\}$, for all $x, y\in R$

(vi). $$(\mu\cup\lambda)(x\wedge y) = \max\{\mu(x\wedge y), \lambda(x\wedge y)\}$$
$$\le \max\{\max\{\mu(x), \mu(y)\}, \max\{\lambda(x), \lambda(y)\}\}$$
$$= \max\{\max\{\max\{\mu(x), \mu(y)\}, \lambda(x)\}, \lambda(y)\}$$
$$= \max\{\max\{\max\{\mu(x), \lambda(x)\}, \mu(y)\}, \lambda(y)\}$$
$$= \max\{\max\{\mu(x), \lambda(x)\}, \max\{\mu(y), \lambda(y)\}\}$$
$$= \max\{(\mu\cup\lambda)(x), (\mu\cup\lambda)(y)\}$$

Therefore, $(\mu\cup\lambda)(x\wedge y) \le \max\{(\mu\cup\lambda)(x), (\mu\cup\lambda)(y)\}$, for all $x, y\in R$

Thus, $\mu\cup\lambda$ is an T-anti-fuzzy ideal of a ℓ-near-ring R.

Theorem: 4

The join of a family of T-anti-fuzzy ideal of ℓ-near-ring R is an T-anti-fuzzy ideal of a ℓ-near-ring R.

Proof:

Let $\{v_\alpha : \alpha\in I\}$ be a family of T-anti-fuzzy ideal of ℓ-near-ring R

Let $V = \vee_{\alpha\in I} V_\alpha$ and let x and y in R.

(i). $$\mu_V(x-y) = T(\mu_V(x-y), \mu_V(x-y))$$
$$\le T(T(\mu_V(x), \mu_V(y)), T(\mu_V(x), \mu_V(y)))$$
$$= T(T(\mu_V(x), \mu_V(y))) = T(\mu_V(x), \mu_V(y))$$

Therefore, $\mu_V(x-y) \le T(\mu_V(x), \mu_V(y))$, for all $x, y\in R$

(ii). Since $\mu(y+x-y) \le \mu(x)$

$$\mu_V(y+x-y) = T\left(\mu_V(y+x-y), \mu_V(y+x-y)\right)$$
$$\le T\left(\mu_V(x), \mu_V(x)\right) = \mu_V(x)$$

Therefore, $\mu_V(y+x-y) \le \mu_V(x)$, for all $x, y \in R$

(iii). Since $\mu(xy) \le \mu(x)$ and $\mu(xy) \le \mu(y)$

$$\mu_V(xy) \le T\left(\mu_V(xy), \mu_V(xy)\right) = T\left(\mu_V(x), \mu_V(x)\right) = \mu_V(x)$$

Therefore, $\mu_V(xy) \le \mu_V(x)$, for all $x, y \in R$

(iv) Since $\mu((x+z)y - x\,y) \le \mu(z)$

$$\mu_V((x+z)y - xy) = T\left(\mu_V((x+z)y - xy), \mu_V((x+z)y - xy)\right)$$
$$\le T\left(\mu_V(z), \mu_V(z)\right)$$
$$\le \mu(z)$$

Therefore, $\mu_V((x+z)y - x\,y) \le \mu_V(z)$, for all $x, y \in R$

(v)
$$\mu_V(x \vee y) = T\left(\mu_V(x \vee y), \mu_V(x \vee y)\right)$$
$$\le T\left(T\left(\mu_V(x), \mu_V(y)\right), T\left(\mu_V(x), \mu_V(y)\right)\right)$$
$$= T\left(T\left(\mu_V(x), \mu_V(y)\right)\right) = T\left(\mu_V(x), \mu_V(y)\right)$$

Therefore, $\mu_V(x \vee y) \le T\left(\mu_V(x), \mu_V(y)\right)$, for all $x, y \in R$

(vi)
$$\mu_V(x \wedge y) = T\left(\mu_V(x \wedge y), \mu_V(x \wedge y)\right)$$
$$\le T\left(T\left(\mu_V(x), \mu_V(y)\right), T\left(\mu_V(x), \mu_V(y)\right)\right)$$
$$= T\left(T\left(\mu_V(x), \mu_V(y)\right)\right) = T\left(\mu_V(x), \mu_V(y)\right)$$

Therefore, $\mu_V(x \wedge y) \le T\left(\mu_V(x), \mu_V(y)\right)$, for all $x, y \in R$

Thus, the join of a family of T-anti-fuzzy ideal of ℓ-near-ring R is an T-anti-fuzzy ideal of a ℓ-near-ring R.

Theorem: 5

The union of a family of T-anti-fuzzy ideal of ℓ-near-ring R is an T-anti-fuzzy ideal of a ℓ-near-ring R.

Proof:

Let $\{U_\alpha : \alpha \in I\}$ be a family of T-anti-fuzzy ideal of R.
Let $A = \bigcup_{\alpha \in I} U_\alpha$ and Let x, y, z in R.

(i)
$$\mu_A(x-y) = \max\left\{\mu_A(x-y), \mu_A(x-y)\right\}$$
$$\le \max\left\{\max\left\{\mu_A(x), \mu_A(y)\right\}, \max\left\{\mu_A(x), \mu_A(y)\right\}\right\}$$

$$= \max\left\{\max\left(\mu_A(x),\, \mu_A(y)\right)\right\}$$
$$= \max\left\{\mu_A(x),\, \mu_A(y)\right\}$$

Therefore, $\mu_A(x-y) \leq \max\left\{\mu_A(x),\, \mu_A(y)\right\}$, for all $x, y \in R$.

(ii) Since $\mu(y+x-y) \leq \mu(x)$

$$\mu_A(y+x-y) = \max\left(\mu_A(y+x-y),\, \mu_A(y+x-y)\right)$$
$$\leq \max\left(\mu_A(x),\, \mu_A(x)\right)$$
$$= \mu_A(x)$$

Therefore, $\mu_A(y+x-y) \leq \mu_A(x)$, for all $x, y \in R$.

(iii) Since $\mu(xy) \leq \mu(x)$ and $\mu(xy) \leq \mu(y)$

$$\mu_A(xy) \leq \max\left\{\mu_A(xy),\, \mu_A(xy)\right\}$$
$$\leq \max\left\{\mu_A(x),\, \mu_A(x)\right\} \leq \mu_A(x)$$

Therefore, $\mu_A(xy) \leq \mu_A(x)$, for all $x, y \in R$.

(iv) Since $\mu((x+z)y - x\,y) \leq \mu(z)$

$$\mu_A((x+z)y - x\,y) = \max\left(\mu_A((x+z)y - x\,y),\, \mu_A((x+z)y - x\,y)\right)$$
$$\leq \max\left(\mu_A(z),\, \mu_A(z)\right) \leq \mu_A(z)$$

Therefore, $\mu_A((x+z)y - x\,y) \leq \mu_A(z)$, for all $x, y \in R$.

(v) $$\mu_A(x \vee y) = \max\left\{\mu_A(x \vee y),\, \mu_A(x \vee y)\right\}$$
$$\leq \max\left\{\max\left\{\mu_A(x),\, \mu_A(y)\right\}, \max\left\{\mu_A(x),\, \mu_A(y)\right\}\right\}$$
$$= \max\left\{\max\left\{\mu_A(x),\, \mu_A(y)\right\}\right\}$$
$$= \max\left(\mu_A(x),\, \mu_A(y)\right)$$

Therefore, $\mu_A(x \vee y) \leq \max\left\{\mu_A(x),\, \mu_A(y)\right\}$, for all $x, y \in R$.

(vi) $$\mu_A(x \wedge y) = \max\left\{\mu_A(x \wedge y),\, \mu_A(x \wedge y)\right\}$$
$$\leq \max\left\{\max\left\{\mu_A(x),\, \mu_A(y)\right\}, \max\left\{\mu_A(x),\, \mu_A(y)\right\}\right\}$$
$$= \max\left\{\max\left\{\mu_A(x),\, \mu_A(y)\right\}\right\}$$
$$= \min\left\{\mu_A(x),\, \mu_A(y)\right\}$$

Therefore, $\mu_A(x \wedge y) \leq \max\left(\mu_A(x),\, \mu_A(y)\right)$, for all $x, y \in R$.

Thus union of a family of T-anti-fuzzy ideal of ℓ-near-ring R is an T-anti-fuzzy ideal of a ℓ-near-ring R.

2 Conclusion

In this paper, we gave some new idea of T-anti-fuzzy ideal of a ℓ-near-ring and use some properties of T-anti-fuzzy ideals of ℓ-near-ring. We hope that our study contributes to the development of these results by other researchers.

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Competing Interests

Authors have declared that no competing interests exist.

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