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Application of the Laplace Decomposition Method for Motion of Spherical/Non-Spherical Particles within a Highly Viscous Fluid

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Abstract

Downward movement of solid particles within a fluid in the presence of a gravitational field occurs in many industrial and engineering processes, e.g. particulate processing and two phase solid-liquid applications. Three highly viscous liquids including water, glycerin and ethylene-glycol were selected to study the motion of spherical/non-spherical solid particles for a wide range of Reynolds numbers employing a drag coefficient as defined by Chien [10]. The governing equation of the motion is strongly nonlinear due to the nonlinear nature of the drag force exerted on the solid body during falling. In this paper, a numerical technique, namely the Laplace Decomposition Method (LDM), is applied to solve the governing equation. This method applies the Laplace transform to the differential equation whereas the nonlinear term is decomposed in terms of Adomian polynomials. A good agreement was achieved when compared with a famous numerical method, then the effects of solid sphericity were tested for the different liquids. This study demonstrates the effectiveness of the present mathematical technique and illustrates a simple application for this type of problem which may be used for a large class of nonlinear differential equations.

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Keywords: Laplace decomposition method, motion of particle, spherical/non-spherical immersed body.

Nomenclature

A_n	*Adomian Decomposition*
a_1,a_2,a_3,a_4	*General Equation Constants*
c	*Integration Constant*
C_D	*Drag Coefficient [-]*
D	*Particle Diameter [m]*
F_B	*Buoyancy Force [N]*
F_D	*Drag Force [N]*
Fg	*Gravitation Force [N]*
F_{VM}	*Virtual Mass Force [N]*
g	*Gravity [m/s^2]*
m	*Mass of Particle [Kg]*
t	*Time [s]*
U	*Terminal Velocity [m/s]*
u	*Velocity Function [m/s]*
Re	*Reynolds Number [-]*
Greek Letters	
φ	*Sphericity[-]*
μ	*Viscosity [Pa.s]*
ρ	*Density [Kg/m^3]*
Subscripts	
f	*Fluid*
s	*Solid*

1 Introduction

There are many conditions where a solid particle falling within a fluid may occur. For instance, petroleum engineering applications, unsteady predictions about liquid-solid mixtures (these predictions may relate to terminal velocity or acceleration of a particle falling within a fluid.), fixed/fluidized bed reactors, etc. For applications such as these, it may be desirable to track the trajectory of a moving particle for the propose of an industrial operation or chemical process [1]. There are two important design parameters for the mentioned applications: 1) terminal velocity of the particle settling through a fluid and 2) drag force exerted on the falling solid body due to the viscosity of the fluid. The drag coefficient which determines the amount of drag force, has undergone extensive investigation during past decades. These studies evaluated the amount of drag coefficient in terms of the Reynolds number, *Re,* for spherical solids. A comprehensive review of previous works may be found in review papers by Khan and Richardson [2], Chhabra [3], Clift et al. [4] and Hartman and Yutes [5].

It is important to note, however, that most real cases are related to non-spherical solid particles, the drag coefficient should be evaluated for such solid particles because drag force is affected by the particle shape and its orientation. Less information exists about drag coefficient of non-spherical particles with respect to spherical ones. Available predictions may be found in [6-8].

Haider and Levenspiel [9] presented a predicting correlation for drag coefficient covering both spherical and non-spherical solids in terms of *Re* and sphericity, φ:

$$C_D = \frac{24}{\mathrm{Re}}\left(1 + e^{(2.3288-6.4581\varphi+2.4486\varphi^2)}\,\mathrm{Re}^{(0.0964+0.5565\varphi)}\right) + \frac{73.69e^{-0.0548\varphi}\,\mathrm{Re}}{\mathrm{Re}+5.738e^{6.2122\varphi}} \quad (1)$$

where *Re*, C_D are:

$$\mathrm{Re} = \frac{UD}{\mu} \quad (2)$$

$$C_D = \frac{F_D}{\frac{\pi}{8}\rho U D^2} \quad (3)$$

F_D illustrates drag force exerted on the solid body. φ is sphericity which denotes the amount of the surface of a sphere having the same volume as the particle to the actual surface area of the particle. In other words, sphericity is a quantity for the value of a solid particle's deviation from a spherical configuration. The more the particle departs from a spherical shape the more the unit aspect ratio is lower than 1(φ is 1 for a true spherical shape.). After Haider and Levenspiel, other researchers suggested a more exact correlation [10,11]. Chein [10] showed that the following relation can predict the drag coefficient more precisely using the concept of sphericity:

$$C_D = \frac{30}{\mathrm{Re}} + 67.289e^{-5.03\varphi} \quad (4)$$

In this study we use Eq. 4 for the drag coefficient value, because it is precise and in a simple form which can be employed in differential equations.

The nonlinear governing equation which will be derived in the next section is solved using the Laplace Decomposition Method (LDM). This technique was first proposed by Adomian [12-13] and is appropriate for a wide range of nonlinear differential equations. This technique assumes an infinite solution in the form $u = \sum_{n=1}^{\infty}(u_n)$, then the Laplace transform is applied to the differential equation. Adomian polynomials are then used for the nonlinear terms and after their decomposition, an iterative manner is created to find the u. This technique was investigated by other researchers [14-18] and their results showed that the exact solution is approximated with a high degree of accuracy using only the few terms of the scheme. This paper shows the application of this technique on the strongly nonlinear governing equation for motion of spherical/non-spherical solid particles within highly viscous fluids. The results are then compared with the Runge-Kutta^{4th} which is applicable for this kind of ODE.

2 Problem Description

Assume a solid particle with the sphericity of φ falling from rest through a Newtonian and viscous fluid which is initially quiescent. The particle will accelerate under the gravity field until it reaches a constant velocity, namely "Terminal Velocity". At this stage, the forces applied to the body, including Buoyancy force, Resistance force, Gravitational force and Virtual mass force, will be balanced, thus no acceleration will influence the body. Fig. 1 indicates a schematic of the present problem and the forces exerted on the solid particle. Note that the resistance force is dependent on the drag coefficient (Eq. 4), and Gravitational force and Buoyancy force are related to the characteristics of the body and the liquid. Added mass force should be taken into account as the fluid around the solid body has acceleration.

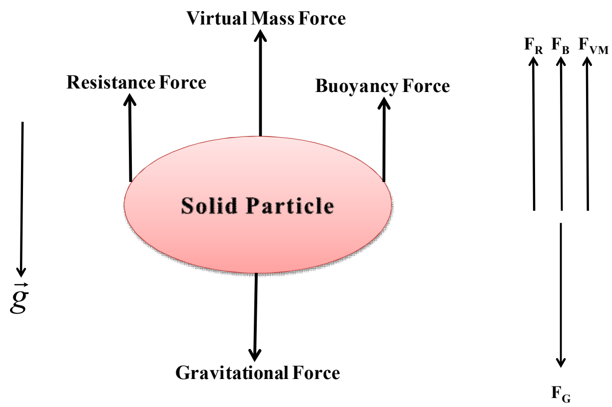

Fig. 1. Schematic of the forces excreted to the solid particle

There are a few simplifying assumptions that make the study easier:

1) No mass transfer between the two phases, so the mass of the particle and the liquid are constant during the falling process.
2) The wall confining effect is negligible.
3) Rectilinearly falling is assumed.
4) The drag force exerted on the solid particle is similar to that applied under constant velocity movement.
5) The Basset force is negligible as the particle is much denser than the liquid.

Under the above considerations, the equation of the motion is as follows [19]:

$$m\frac{du}{dt} = mg(1-\frac{\rho_f}{\rho_s}) - \frac{1}{8}\pi D^2 \rho_f C_D u^2 - \frac{1}{12}\pi D^3 \rho_f \frac{du}{dt} \tag{5}$$

Where the left-side of the equation represents Newton's dynamics rule and the right-side terms are representing Buoyancy force, Drag force (Resistance force) and the added mass effect. The last term (added mass effect) is due to the acceleration of the fluid around the solid body. The nonlinear term related to the drag force causes the main difficulty in the problem. Rearranging the parameters, the following equation with the indicated initial condition is achieved:

$$a_1\frac{du}{dt} + a_2 u + a_3 u^2 - a_4 = 0, \quad u(0) = 0 \ ; \tag{6}$$

where the constants of Eq. 6 and the exact solution of Eq. 6 are:

$$a_1 = \left(m + \frac{1}{12}\pi D^3 \rho_f \right)$$
$$a_2 = 3.75\pi D\mu$$
$$a_3 = \frac{67.289e^{(-5.03\varphi)}}{8}\pi D^2 \rho_f$$
$$a_4 = mg(1 - \frac{\rho_f}{\rho_s})$$
$$u(t) = \frac{1}{2}\frac{-a_2 + \tanh(\frac{1}{2}\frac{\sqrt{4a_3a_4 + a_2^2}}{a_1}(t+c))\sqrt{4a_3a_4 + a_2^2}}{a_3}$$

LDM, which is applicable for a wide range of nonlinear ODE problems, is used to solve Eq. 6. This method is described in the next section.

3 Application of LDM

Consider the nonlinear equation of Eq. 6 with the specified initial conditions. First, LDM applies the Laplace transformation to Eq. 6.

$$\ell\ [a_1 \frac{du}{dt}] + \ell\ [a_2 u] + \ell\ [a_3 u^2] - \ell\ [a_4] = 0; \tag{7}$$

where ℓ denotes the Laplace transformation operator. Applying the property of Laplace transformation and using the initial boundary condition mentioned in Eq. 6:

$$a_1 S\ell\ [u] + a_2 \ell[u] + a_3 \ell\ [u^2] - \frac{a_4}{S^2} = 0; \tag{8}$$

assumes the solution as an infinite series:

$$u = \sum_{n=0}^{\infty} u_n; \tag{9}$$

where the term u_n is to be recursively computed. The nonlinear term is decomposed as:

$$f(u) = u^2 = \sum_{n=0}^{\infty} A_n; \tag{10}$$

where $A_n = A_n(u_0, u_1, u_2, \ldots, u_n)$ are the so called Adomian polynomials [20] given as:

$$A_n = \frac{1}{n!}\frac{d^n}{d\eta^n}[N(\sum_{i=0}^{\infty}\eta^i u_i)]_{\eta=0} \quad , \quad n = 0,1,2,... \tag{11}$$

So we have:

$$\begin{aligned} A_0 &= f(u_0) = u_0^2 \\ A_1 &= u_1 f'(u_0) = 2u_0 u_1 \\ A_2 &= u_2 f'(u_0) + \frac{1}{2!}u_1^2 f''(u_0) = 2u_0 u_2 + u_1^2 \\ A_3 &= u_3 f'(u_0) + u_1 u_2 f''(u_0) + \frac{1}{3!}u_1^3 f'''(u_0) = 2u_0 u_3 + 2u_1 u_2 \\ &. \\ &. \\ &. \end{aligned} \tag{12}$$

Substituting Eqs. 9 to 12 into Eq. 8 results in:

$$\ell\ [\sum_{n=0}^{\infty} u_n] = -\frac{a_2}{a_1 S}\ell\ [\sum_{n=0}^{\infty} u_n] + \frac{a_4}{a_1 S^2} - \frac{a_3}{a_1 S}\ell\ [\sum_{n=0}^{\infty} A_n]; \tag{13}$$

or

$$\ell\ (u_0 + u_1 + u_2 + ...) = -\frac{a_2}{a_1 S}\ell(u_0 + u_1 + u_2 + ...) + \frac{a_4}{a_1 S^2} - \frac{a_3}{a_1 S}\ell\ (A_0 + A_1 + A_2 + ...); \tag{14}$$

Matching both sides of Eq. 14 and applying the inverse Laplace transformation yields the following iterative algorithm:

$$\begin{aligned} &\ell\ (u_0) = \frac{a_4}{a_1 S^2} \Rightarrow u_0 = \frac{a_4}{a_1}t\ ; \\ &\ell\ (u_1) = -\frac{a_2}{a_1 S}\ell\ (u_0) - \frac{a_3}{a_1 S}\ell\ (A_0) \Rightarrow u_1 = -\frac{1}{2}\frac{a_2 a_4 t^2}{a_1^2} - \frac{1}{3}\frac{a_3 a_4^2 t^3}{a_1^3}; \\ &\ell\ (u_2) = -\frac{a_2}{a_1 S}\ell\ (u_1) - \frac{a_3}{a_1 S}\ell\ (A_1) \Rightarrow u_2 = \frac{1}{30}\frac{a_4 t^3 (5a_2^2 a_1^2 + 10\ a_1 a_2 a_3 a_4 t + 4a_3^2 a_4^2 t^2)}{a_1^5}; \\ &\ell\ (u_3) = -\frac{a_2}{a_1 S}\ell\ (u_2) - \frac{a_3}{a_1 S}\ell\ (A_2) \Rightarrow u_3 = -\frac{1}{2520}\frac{a_4 t^4 (105\ a_2^3 a_1^3 + 462\ a_1^2 a_2^2 a_3 a_4 t)}{a_1^7} \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad - \frac{1}{2520}\frac{(476\ a_1 a_2 a_3^2 a_4^2 t^2 + 136\ a_3^3 a_4^3 t^3)}{a_1^7} \\ &. \\ &. \\ &. \end{aligned} \tag{15}$$

Other terms of u_4, u_5 ... can be obtained recursively in a similar fashion. As u is the velocity of the solid particle, we can integrate a combination of the few terms generated by Eq. 15 to get a relation for the displacement of the particle assuming a zero value for its initial movement. Also, differentiating those terms of Eq. 15 will result in a governing equation for the acceleration of the particle, with an earth gravitation value (9.81) for the initial acceleration.

4 Results and Discussion

The previous section described the solution method for the purely non linear equation, Eq. 6. It is clear that to get closer to the exact solution, a higher number of terms than generated in Eq. 15 is desired. To examine the convergence and accuracy of the method, consider a specific problem of Eq. 6, where all coefficients are unity:

$$\frac{du}{dt} + u + u^2 - 1 = 0, \quad u(0) = 0 \ ; \tag{16}$$

Now using the 3 to 5 terms obtained in Eq. 15 and substituting unity into all coefficients, the results can be compared with each other in Fig. 2. The comparison between the results reveals that the five term solution of LDM can predict the Runge-Kutta ^{4th} solution well, but to enhance the accuracy and convergence of the solution we use a seven term solution of LDM for the rest of results.

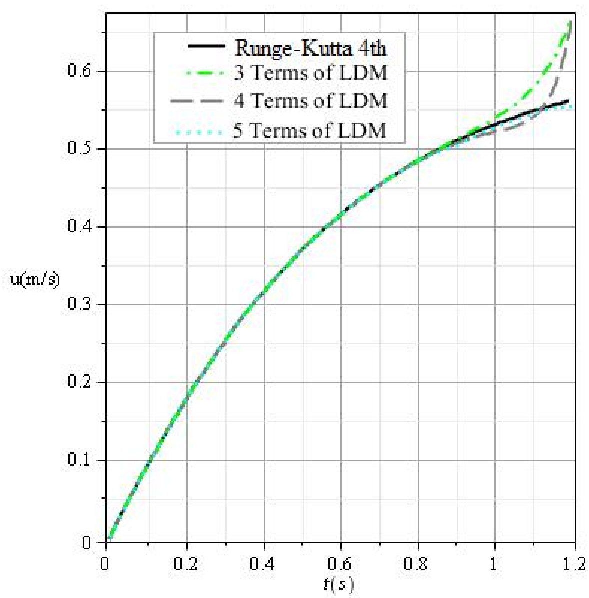

Fig. 2. LDM solution for different 3,4 and 5 terms in comparison with the Runge-Kutta solution

In Table 1, results obtained from other techniques and the Runge-Kutta solution are compared with the values obtained using LDM. The comparison between the results reveals that the present technique predicts the 4th Runge-Kutta solution with high accuracy and using higher orders of the LDM technique leads to lower relative error. Now we substitute real values for the coefficients used in Eq. 6.

Table 2 shows the properties of the real condition where a solid particle is falling within three highly viscous fluids including water, ethylene glycol and glycerin. A solid aluminum particle has a density of 2702 Kg/m^3. The velocity profiles of a particle falling within the mentioned fluids are depicted for four sphericities in Fig. 3. This figure shows the velocity magnitude vs. time for different values of sphericity and liquids. In the initial stage of falling, the velocity magnitude increases until it reaches the constant speed, or "Terminal Velocity". Due to the higher viscosity of glycerin, the falling solid reaches terminal velocity faster in glycerin with respect to the other liquids. Also the acceleration duration is increased by the reduction of liquid viscosity.

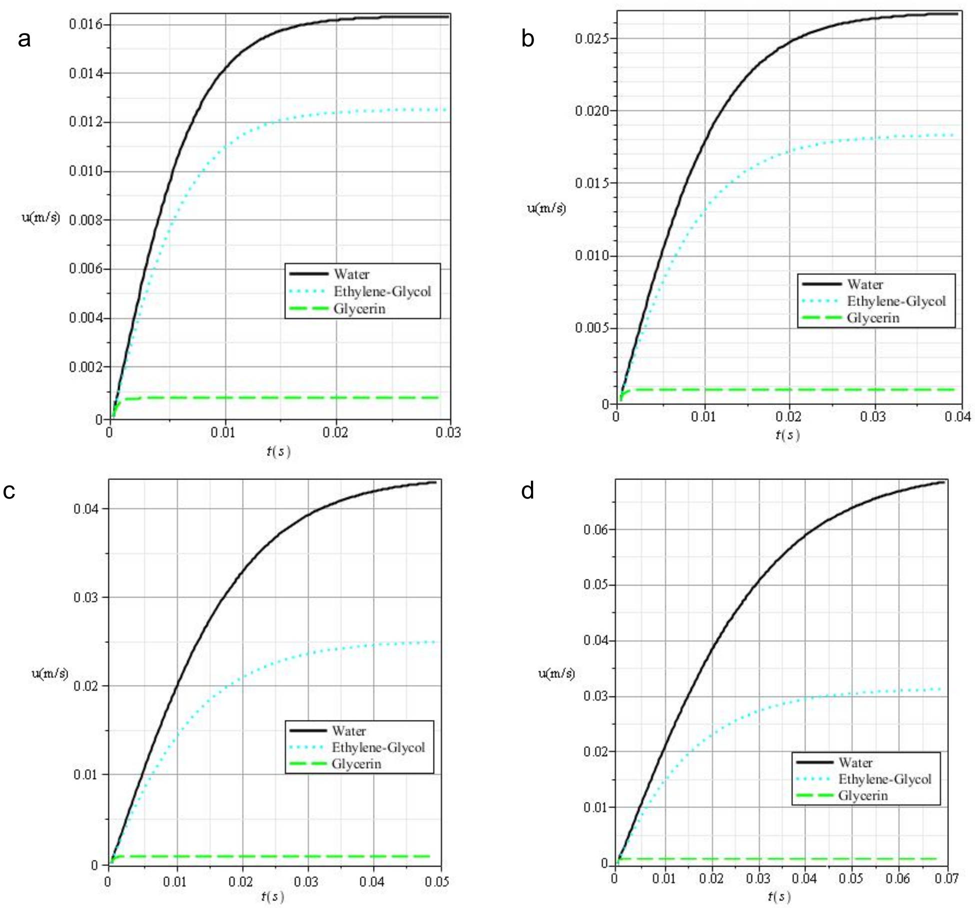

Fig. 3. Velocity profiles for 3 liquids and spherecity values of a) 0.2 b) 0.4 c) 0.6 d) 0.8

The effect of spherecity of the solid on the velocity profile is illustrated in Figs. 4 to 6. With the increase of sphericity the value of the terminal velocity is enhanced. For high viscous liquids such as glycerin, the effect of sphericity is not significant and can be neglected for particle applications as depicted in Fig. 4. Acceleration profiles of the particle are shown in Figs. 7 to 9. Results demonstrate that a particle with higher sphericity brings higher acceleration duration and displacement. Furthermore, for lower liquid viscosity, the acceleration magnitude of the particle is higher.

Using definite and indefinite integration for the terms evaluated in Eq. 16, the displacement governing equation can be found. Fig. 10 depicts the schematic of the displacement for a solid particle in different conditions. With the increase of liquid viscosity, the falling velocity magnitude of the solid particle is reduced considerably and the particle transverses a smaller height of the liquid column, also particle with a larger sphericity moves faster than a smaller one.

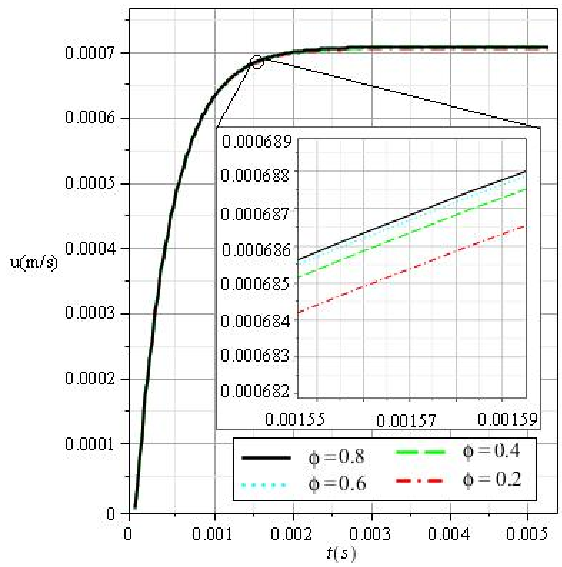

Fig. 4. Velocity profiles of different solid spherecity used in Glycerin.

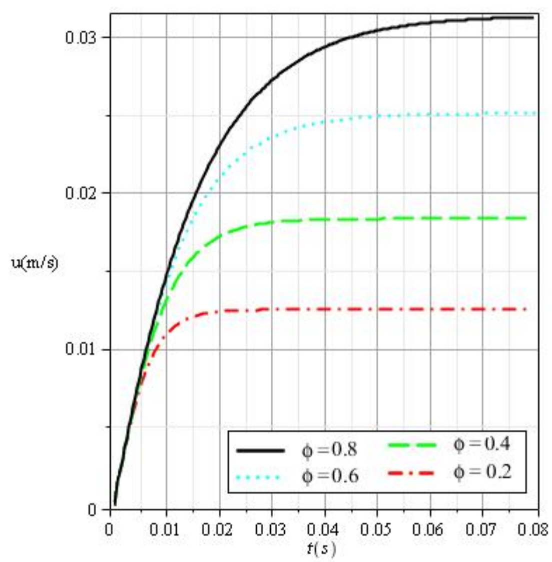

Fig. 5. Velocity profiles of different solid spherecity used in Ethylene glycol

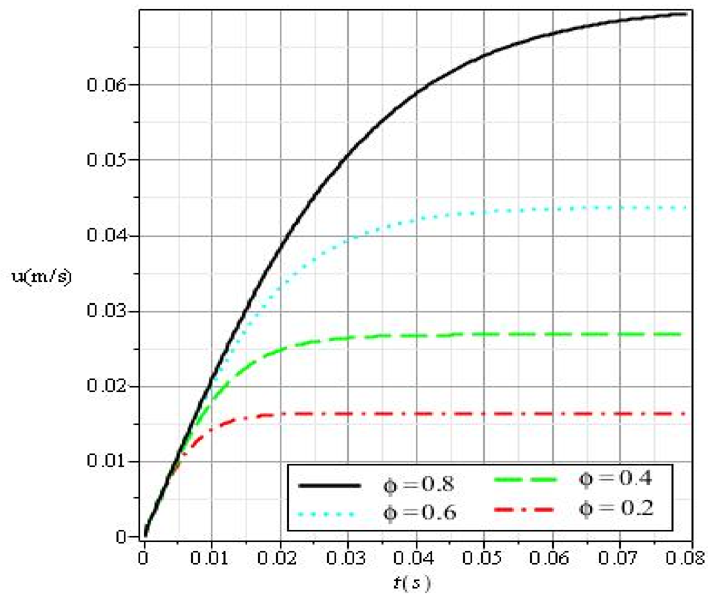

Fig. 6. Velocity profiles of different solid spherecity used in Water

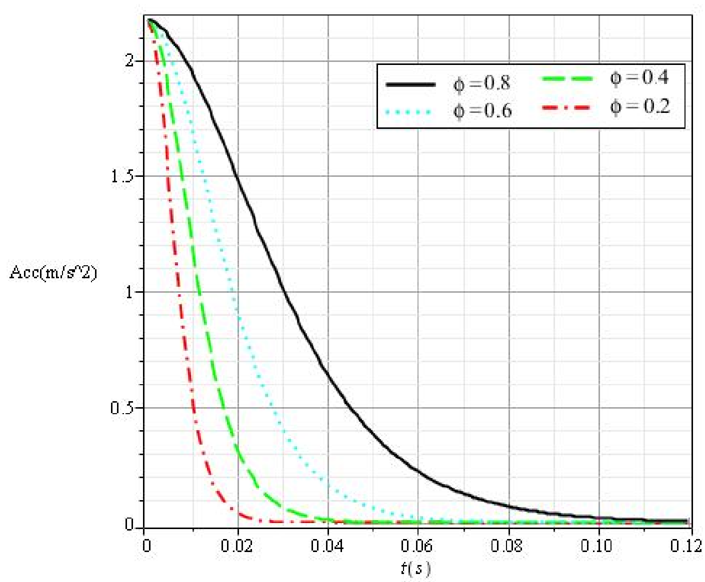

Fig. 7. Acceleration profiles of different solid spherecity used in Water

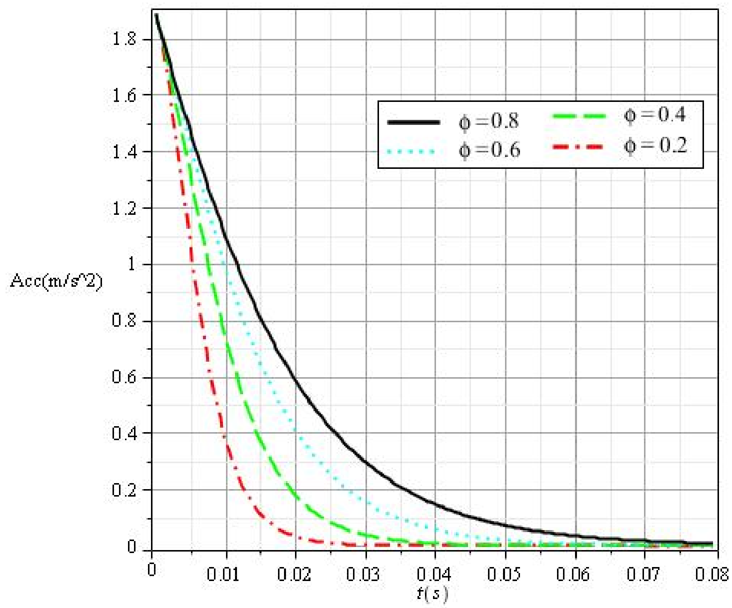

Fig. 8. Acceleration profiles of different solid spherecity used in Ethylene glycol

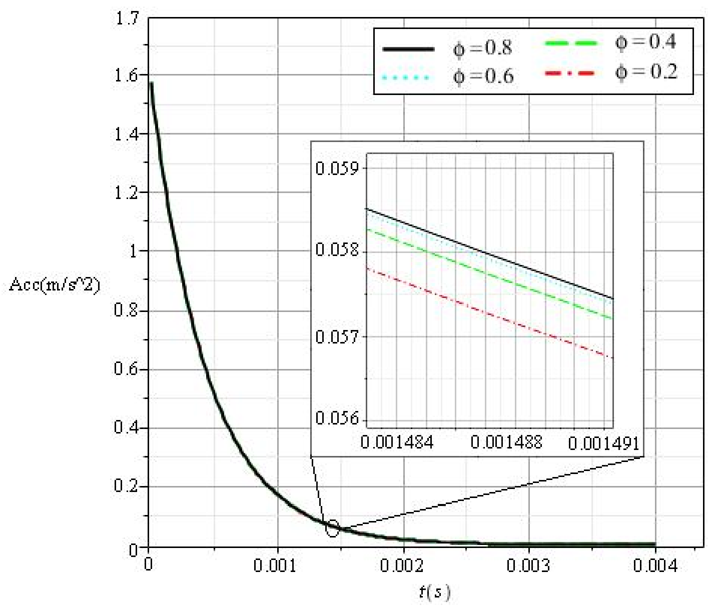

Fig. 9. Acceleration profiles of different solid spherecity used in Glycerin

Table 1. Comparison of the results obtained from different techniques with LDM terms for Eq. 16

t	VIM [19]	HPM [21]	LDM (5 terms)	Exact solution	VIM relative Er%	HPM relative Er%	LDM relative Er%
0.1	0.0948517	0.0948619	0.0948619	0.0948619	0.0107	≈ 0	≈ 0
0.2	0.1791031	0.1791145	0.1791133	0.1791133	0.0057	0.0006	≈ 0
0.3	0.2526879	0.2526896	0.2526907	0.2526907	0.0011	0.0004	≈ 0
0.4	0.3160149	0.3160075	0.3160070	0.3160070	0.0025	0.0001	≈ 0
0.5	0.3697895	0.3698068	0.3698062	0.3698063	0.0045	0.0001	≈ 0
0.6	0.4150429	0.4150405	0.4150305	0.4150282	0.0035	0.0029	0.000054
0.7	0.4527300	0.4527300	0.4527298	0.4526970	0.0072	0.0072	0.000072

Table 2. Properties of fluids and related coefficients of Eq. 6

Fluid	Density (Kg/m^3)	Viscosity (Pa.s)	a_1	a_2	$a_3/\exp(-5.03\varphi)$	a_4
Water	996.51	0.001	0.000010863706	0.000035341875	0.236981979	0.000023665598
Ethylene-glycol	1111.40	0.0157	0.000011675791	0.000554867437	0.264304193	0.000022071370
Glycerin	1259.90	0.799	0.000012725445	0.02823815812	0.299619266	0.000020010765

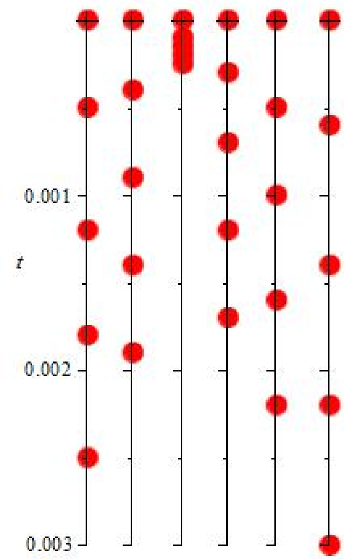

Fig. 10. Displacement schematic for a) ϕ =0.5, Water b) ϕ =0.5, Ethylene glycol c) ϕ =0.5, Glycerin d) ϕ =0.3, Ethylene glycol e) ϕ =0.6, Ethylene glycol f) ϕ =0.9, Ethylene glycol.

5 Conclusion

In this study, the Laplace Decomposition Method (LDM) is employed to obtain the solution for a nonlinear governing equation of a falling spherical/non spherical solid particle within highly viscous liquids using data available in petroleum engineering and processing literature. Firstly, the general equation is derived, then real condition is assumed to obtain the related coefficients of the purely nonlinear equation. Comparison of this method and other techniques revealed a good agreement with the numerical solution, Runge-Kutta ^{4th}, and high accuracy of this method even with fewer terms. Outcomes were reported in terms of velocity, acceleration and displacement profiles. The present work approved the simplicity and capability of LDM and explained an exact expression, especially for the initial stages, which is valuable for engineering problems in the field of chemistry and powder technology. This technique could be employed in a wide range of two-phase systems problems.

Competing Interests

Authors have declared that no competing interests exist.

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