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Outage Analysis of a Multi-User Spatial Diversity System in a Shadow-Fade Propagating Channel

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Author's contribution

This work was carried out by author VE. He designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. He has read and approved the final manuscript.

Research Article

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ABSTRACT

In a wireless network, communication between a source and a destination mobile station (DMS) fails to establish if the source or the DMS is located inside a deep shadow-fading propagating channel. In this situation, intermediate mobile stations may be used to relay the signal between the two nodes. In a cellular system the source is a base station (BS) and the DMS is a weak mobile station (MS) while in an ad-hoc network, the source and the DMS are two nodes of the network. This paper presents the scheme of "multi-user spatial diversity" as a method of diversity to combat the undesired shadow-fade channel behavior. A model is presented for the case where one or several mobile stations (MSs) relay the signal between the source and the DMS, in a shadow-fading environment. A formula is derived for the average outage probability of the received signal-to-noise ratio at the DMS, when M intermediate MSs relay the signal from the source to the DMS according to a particular protocol. The outage probability improves as the number of the relays increases.

Keywords: Multi-user spatial diversity; Ad-hoc networks; wireless cellular communications; outage probability; independent shadowing; relay mobile station; macro diversity.

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1. INTRODUCTION

Spatial diversity (using multiple antennas at the transmitter and/or receiver) is a common method of combating the deleterious effects of fading and shadowing in a wireless communication network [1-2]. This method, however, has usually been associated with point-to-point links. Recently the work of [3-7] has suggested the idea of using a possible idle mobile station (MS) as a relay between a source and a destination mobile station (DMS) to envisage a new type of (mobile-assisted) spatial diversity that may be classified as distributed spatial diversity 5].

Some form of collaboration among users is needed in order to implement distributed spatial diversity. In 6] the users select one partner from the in-cell users. Each user decodes the information symbols of its partner and transmits them along with its own information. Consequently, the user achieves spatial diversity by using its partner's antenna. It turns out that even though the inter-user channel is noisy, the set of achievable rates for a system with two cooperative users is a superset of the capacity region obtained without cooperation. The idea of collaboration among MSs in a cellular system is proposed in 3,5]. In the scheme that is discussed in these references, an MS retransmits a signal from the base station (BS) to a nearby MS that is in a shadowed and faded region relative to the BS. In 3], the concept of shadow-fade environment is described in details. In the analysis provided in [3,5] the additional noise at the relay was neglected and the gain of the amplifier at the relay was independent of the received signal's power. Theoretical and simulation results in [3,5] show substantial reduction of the outage probability for a collaborative system, in comparison with that of a non-collaborative system using 2-transmit antenna micro-diversity in a composite lognormal-Rayleigh shadowing-fading environment.

The work in2,6,7,8] analyzes the performance of a system with a relay that decodes and forwards its partner's information symbols versus a system with a relay that only amplifies the received signal. For independent Gaussian flat fading channels with path loss, it is argued in [7], by simulations, that the less complex cooperative system with an amplifying relay has a similar average bit error rate as the system using a decoding relay. To confirm the good performance of the amplify-and-forward strategy, the outage probability of the achievable rate in a two steps collaborative system using orthogonal channels (e.g., an Frequency Division Multiple Access or FDMA system) is computed in [5]. For an amplifying relay, it is shown that this outage probability is comparable to the outage probability of the same system that is optimized to select among: no relay, amplifying relay, and decoding relay, depending on the reliability of the connection between source and relay. Furthermore, it is argued in [8] that the amplify-and-forward strategy slightly loses performance from the case of the ideal transmit diversity in which the 2 transmit antennas are located at the source (i.e., noiseless communication between source and relay). Related studies to this work are further presented in [9-19].

The system presented in this paper is depicted inFig. **1**. When the communication between the source and the DMS is affected by a deep shadow-fade situation, the source is allowed to use other MSs as relays, to communicate with the DMS. Let's call an intermediate MS a relay MS or RMS. Since the relay channels are often independent, multiple independently faded-shadowed versions of the transmitted signal are received, thereby providing diversity transmission.

British Journal of Applied Science & Technology, 4(1): 40-53, 2014

Fig. 1. Multi-user spatial diversity in a wireless network. Nearby MSs (RMSs) are employed to relay the signal to the destination mobile station (DMS)

Because usually more than one MS can be involved in relaying the signal, let's call such diversity "multi-user spatial diversity". The major contributions of the paper are as follows:

- 1. A formula is derived for the outage probability of the received signal-to-noise ratio (SNR) at the DMS, when *M* RMSs are used as relays between the source and the DMS, assuming the receiver at the DMS uses selection combining and the channels incur path loss and independent multipath flat fading-lognormal shadowing.
- 2. A complete discussion of multi-user spatial diversity and the associated signal models in a wireless network are presented.
- 3. The distribution of the received Signal to Noise Ratio (SNR) and the outage probability at DMS are derived, when there is a constraint on the transmit power at all relays (i.e. they are equipped with automatic gain control (AGC)). The reason for using AGCs is that the relays are indeed MSs with limited power. This power is used for the sake of other MSs, and one may not want to share more than a specific amount of its power for relaying purposes.
- 4. The paper also presents results for the case in which the additional noise in the relay is ignored. This yields a simpler closed-form formula for the outage probability of the received SNR. The paper also includes the results of a study into the probability of restoring communication between the source and the DMS as a function of the number of RMSs.

2. MULTI-USER SPATIAL DIVERSITY

Let's define user diversity to be a form of spatial diversity in which a signal is relayed, through an independent channel, between the source and the DMS by another MS in the network. In the method analyzed in this paper, multiple users are involved in relaying the signal between the source and the DMS, leading to the use of the term "multi-user spatial diversity". Only amplification at the relays is considered in this paper.

Fig. 2 shows a system that employs such multi-user spatial diversity, and the channel designation for the various links. In this figure RMS₁, RMS₂, ..., RMS_M are *M* relays *nearby to theDMS*, assumed to be ordered with RMS_1 the nearest MS to the DMS and RMS_2 the second nearest MS to the DMS and so on. When the direct channel from the source to the DMS faces a deep shadow-fade situation, the communication is recovered by employing a relay (the nearest MS i.e. $RMS₁$) between them. If the problem still remains, two relays (the nearest plus the second nearest i.e. $RMS₁$ and $RMS₂$) are employed to reestablish the connection and so on. The source and the DMS are assumed to be communicating while *M* MSs relay the signal between them. Assuming that C₀, C₁, ..C_MareM+1 orthogonal channels, the scheme works as follows. The source broadcasts the signal to the DMS and the RMSs in channel C₀. RMS_m ($m=1,2, ..., M$) converts the data from the channel C₀ to channel C_m, and then amplifies and relays it to the DMS. Therefore, the DMS receives signals from *M*+1 orthogonal channels $(C_0, C_1, \ldots C_M)$.

Fig. 2. *M* **RMSs relay the signal between the source and the DMS.**

The discrete-time baseband equivalent multichannel model consists of *M* +1 sub channels between the source and the DMS

$$
y_0 = h_0 \sqrt{\varepsilon_0} x + z_0
$$

\n
$$
y_m = h_{2,m} \alpha_m \left(h_{1,m} \sqrt{\varepsilon_0} x + z_{1,m} \right) + z_{2,m}, \qquad m \in [1, M],
$$
\n(1)

Wherex is the transmitted signal from the source, y_0 is the received signal from the direct path (source \Rightarrow DMS), and $\ y_{_m}$ is the received signal from the m^{th} relay (RMS $_m\!\!\rightarrow$ DMS). The maximum transmit power from the source is given by ε_{0} . If automatic gain control (AGC) is used at the mth relay, $\alpha_{_m}$ is an amplification factor that ensures the transmit power at the RMS_m falls below a specific value of ε_m . z_0 and $z_{2,m}$ are received AWGN (Additive White Gaussian Noise) , both assumed to have power $\,N_{_0}$. $\,h_{_0}$, $\,h_{_{1,m}}$, and $\,h_{_{2,m}}$ are assumed to be independent and complex Gaussian distributed with zero mean and variance $\Omega_{_0}$, $\Omega_{_{1,m}}$, and $\Omega_{2,m}^{\phantom i}$ respectively. Also $\,z_{1,m}^{\phantom i}$ is the additional noise at the m th relay.

The parameters $\Omega_{_0}$, $\Omega_{_{1,m}}$, and $\Omega_{_{2,m}}$ are assumed to be lognormal with the following PDF (probability density function)

British Journal of Applied Science & Technology, 4(1): 40-53, 2014\n
$$
f_{\Omega_{0}}(\Omega) = \frac{1}{\sqrt{2\pi\sigma_{0}\Omega}}e^{-\frac{(\ln(\Omega)-\mu_{0}(L))^{2}}{2\sigma_{0}^{2}}}, f_{\Omega_{1,n}}(\Omega) = \frac{1}{\sqrt{2\pi\sigma_{1,m}\Omega}}e^{-\frac{(\ln(\Omega)-\mu_{1,n}(L))^{2}}{2\sigma_{1,n}^{2}}},
$$
\n
$$
f_{\Omega_{2,n}}(\Omega) = \frac{1}{\sqrt{2\pi\sigma_{2,m}\Omega}}e^{-\frac{(\ln(\Omega)-\mu_{2,n}(r_{n}))^{2}}{2\sigma_{2,n}^{2}}} \qquad (2)
$$
\nwhere $\mu_{0}(L)$, $\mu_{1,m}(d_{m})$ are the area mean powers for the distances L and d_{m} of the source to the DMS and RMS_m respectively, and $\mu_{2,m}(r_{m})$ is the area mean power for the distance r_{m} of the RMS_m to the DMS (
\nFig. 3). These parameters are given by\n
$$
\mu_{0}(L) = \mu - \beta \ln(L/d), \ \mu_{1,m}(d_{m}) = \mu - \beta \ln(d_{m}/d), \ \mu_{2,m}(r_{m}) = \mu - \beta \ln(r_{m}/d),
$$
\nwhere d , μ , and β are the reference distance, the area mean power at the reference distance, and the path loss exponent respectively.\nIn the following sections the outage probabilities of systems employing multi-user spatial simulations in this section, are investigated for flat shadow-fading channels. All the simulations in this paper is performed in MATLAB.

where $\mu_0(L)$, $\mu_{1,m}(d_m)$ are the area mean powers for the distances L and d_m of the distance $r_m^{}$ of the RMS $_m$ to the DMS (Fig. 3). These parameters are given by

$$
\mu_0(L) = \mu - \beta \ln(L/d), \ \mu_{1,m}(d_m) = \mu - \beta \ln(d_m/d), \ \mu_{2,m}(r_m) = \mu - \beta \ln(r_m/d),
$$

distance, and the path loss exponent respectively.

In the following sections the outage probabilities of systems employing multi-user spatial diversity, as defined in this section, are investigated for flat shadow-fading channels. All the simulations in this paper is performed in MATLAB.

Fig. 3. Relative positions of the source, the DMS and the RMS*^m*

3. OUTAGE PROBABILITIES

The outage probability of the received signal at the DMS, when *M* RMSs relay the signal between the source and the DMS according to the protocol described in the previous section is shown here. Outage happens when the received SNR at the DMS falls below a specific threshold. In the first subsection the assumption is that transmit power at RMS*^m* is limited to $\varepsilon_{_{\!m}}$, and in the second subsection a simpler formula is derived for the outage by ignoring the noise at the relays.

3.1 With AGC at Relays

First the probability of outage is calculated at the DMS for the channel 0 (direct path between source and DMS). Using(1), the signal-to-noise ratio for this channel is given by

$$
\gamma_0 = |h_0|^2 \, \varepsilon_0 / N_0 \tag{3}
$$

Since $\left|h_{0}\right|$ has a Rayleigh distribution, it can be shown that $\,{\gamma}_{0}\,$ is exponentially distributed with the following PDF

British Journal of Applied Science & Technology, 4(1): 40-53, 2014

\n
$$
\gamma_0 = \left| h_0 \right|^2 \varepsilon_0 / N_0
$$
\n(3)

\nSince $|h_0|$ has a Rayleigh distribution, it can be shown that γ_0 is exponentially distributed with the following PDF

\n
$$
f_{\gamma_0}(\gamma | \Omega_0) = \frac{1}{\Omega_0} e^{-\frac{\gamma}{\Omega_0}} u(\gamma).
$$
\n(4)

British Journal of Applied Science & Technology, 4(1): 40-53, 2014
 $\gamma_0 = |h_0|^2 c_0 / N_0$ (3)

Since $|h_0|$ has a Rayleigh distribution, it can be shown that γ_0 is exponentially distributed

with the following PDF
 British Journal of Applied Science & Technology, 4(1): 40-53, 2014
 $\int_0^{\infty} = |h_0|^2 \mathcal{L}_0 / N_0$ (3)

ince $|h_0|$ has a Rayleigh distribution, it can be shown that γ_0 is exponentially distributed

ith the following For a given $\, \Omega_{_{0}}$, the probability of $\, \gamma_{_{0}} \,$ being less than a specific threshold (γ) is given by *British Journal of Applied Science & Technology, 4(1): 40-53, 2014*
 $\gamma_0 = |h_0|^2 \varepsilon_0 / N_0$ (3)

Since $|h_0|$ has a Rayleigh distribution, it can be shown that γ_0 is exponentially distributed

with the following PDF British Journal of Applied Science & Technology, 4(1): 40-53, 2014

. (3)
 $\left| h_0 \right|^2 c_0 / N_0$ (3)

. (3)

the following PDF
 $(\gamma | \Omega_0) = \frac{1}{\Omega_0} e^{-\frac{\gamma}{\Omega_0}} u(\gamma)$. (4)

a given Ω_0 , the probability of γ_0 being les is random, the average outage probability is given by on, it can be shown that γ_0 is exponentially distributed

(4)
 γ_0 being less than a specific threshold (γ) is given by

dom, the average outage probability is given by
 $\frac{(\ln(\Omega)-\mu_0(L))^2}{2\sigma_0^2}d\Omega$. (5)

culat ith the following PDF
 $\hat{C}_{\gamma_0}(\gamma | \Omega_0) = \frac{1}{\Omega_0} e^{-\frac{\gamma}{\Omega_0}} u(\gamma)$.

or a given Ω_0 , the probability of γ_0 being less than a specific
 $\Omega_0(\Omega_0) = 1 - e^{-\frac{\gamma}{\Omega_0}}$. Since Ω_0 is random, the average outage pro British Journal of Applied Science & Technology, 4(1): 40-53, 2014
 $[\mathcal{L}_0 / N_0$ (3)

| has a Rayleigh distribution, it can be shown that γ_0 is exponentially distributed
 η_0) = $\frac{1}{\Omega_0} e^{\frac{\gamma}{4\Omega_0}} u(y)$.

(4)
 Eastally $Y_0 = |h_0|^2 \mathcal{E}_0 / N_0$ (3)
 Easta P h_0 and **Example 1 Example 1**
 Easta P P Example 1 Example 1
 Easta P P Exam $\begin{aligned}\n &\int_{0}^{\infty} \mathcal{E}_{0} / N_{0} \quad (3) \\
 &\int_{0}^{\infty} \left| \log \frac{1}{\Omega_{0}} \right|^{2} \text{ is exponentially distribute} \\
 &\int_{\Omega_{0}}^{\infty} \text{ has a Rayleigh distribution, it can be shown that } \gamma_{0} \text{ is exponentially distribute} \\
 &\int_{\Omega_{0}}^{\infty} \text{ following PDF} \quad \text{Ω_{0}, the probability of } \gamma_{0} \text{ being less than a specific threshold }(\gamma) \text{ is given by} \\
 &\int_{\Omega_{0}}^{\infty} \text{Since } \Omega_{0} \text{ is random, the average outage probability is given by}$ Botton Journal of Applied Science & Technology, $4(t)$: 40-53, 2014
 $= |h_0|^2 c_0 / N_0$ (3)
 $\text{ce } |h_0|$ has a Rayleigh distribution, it can be shown that γ_0 is exponentially distributed

the following PDF
 $(y | \Omega_0) = \frac{$ $y_0 = |h_0|^2 \varepsilon_0 / N_0$ (3)

Since $|h_0|$ has a Rayleigh distribution, it can be shown that y_0 is exponentially distributed

with the following PDF
 $f_{p_0}(y | \Omega_0) = \frac{1}{\Omega_0} e^{-\frac{y}{\Omega_0}}$

For a given Ω_0 , the proba

$$
P_0 = E_{\Omega_0}[P(\Omega_0)] = \int_0^\infty \frac{1 - e^{-\frac{\gamma}{\Omega_0}}}{\sqrt{2\pi}\sigma_0 \Omega} e^{-\frac{(\ln(\Omega) - \mu_0(L))^2}{2\sigma_0^2}} d\Omega.
$$
 (5)

Now the probability of outage is calculated at the DMS for channel m (source \rightarrow RMS_m \rightarrow DMS). Assuming that the additional noise at the m^{th} relay is AWGN with power $N^{\vphantom{\dagger}}_0$, the SNR at this relay (RMS*m*) is be probability of outage is calculated at the DMS for channe

Assuming that the additional noise at the m^{th} relay is AWGN w

blay (RMS_m) is
 $h_{1,m} \Big|^2 \varepsilon_0 / N_0$.

gain, $\gamma_{1,m}$ is exponentially distributed with pa

$$
\gamma_{1,m} = |h_{1,m}|^2 \, \varepsilon_0 / N_0 \,. \tag{6}
$$

Once again, $\gamma_{_{1,m}}$ is exponentially distributed with parameter $\Omega_{_{1,m}}$. To satisfy the transmit power constraint at RMS_m, the AGC power gain is¹

$$
\alpha_m^2 = \frac{\varepsilon_m}{\left|h_{1,m}\right|^2 \varepsilon_0 + N_0} \,. \tag{7}
$$

Using (1), the SNR at the DMS is given by

For a given
$$
\Omega_0
$$
, the probability of γ_0 being less than a specific threshold (γ) is given by
\n $P_0(\Omega_0) = 1 - e^{-\frac{\gamma}{\Omega_0}}$. Since Ω_0 is random, the average outage probability is given by
\n
$$
P_0 = E_{\Omega_0}[P(\Omega_0)] = \int_0^\infty \frac{1 - e^{-\frac{\gamma}{\Omega_0}}}{\sqrt{2\pi \sigma_0 \Omega}} e^{-\frac{(ln(\Omega) - \mu_0(L))^2}{2\sigma_0^2}} d\Omega.
$$
\n(5)
\nNow the probability of outage is calculated at the DMS for channel *m* (source \rightarrow RMS_m \rightarrow
\nDMS). Assuming that the additional noise at the *m*th relay is AWGN with power N_0 , the SNR
\nat this relay (RMS_m) is
\n
$$
\gamma_{1,m} = \left| h_{1,m} \right|^2 \varepsilon_0 / N_0.
$$
\n(6)
\nOnce again, $\gamma_{1,m}$ is exponentially distributed with parameter $\Omega_{1,m}$. To satisfy the transmit
\npower constraint at RMS_m, the AGC power gain is¹
\n
$$
\alpha_m^2 = \frac{\varepsilon_m}{\left| h_{1,m} \right|^2 \varepsilon_0 + N_0}.
$$
\nUsing (1), the SNR at the DMS is given by
\n
$$
\gamma_m = \frac{\left| h_{1,m} \right|^2 \left| h_{2,m} \right|^2 \alpha_m^2 \varepsilon_0}{N_0 \left(1 + \left| h_{2,m} \right|^2 \alpha_m^2 \right)}.
$$
\n(8)
\nSubstituting (7)
\nin (8), the following is obtained

Substituting (7)in (8), the following is obtained

¹ The transmit power at the RMSmis ^m . On the other hand from (1) this power is calculated to be. Equation (7) is obtained by equalizing these two.

British Journal of Applied Science & Technology, 4(1): 40-53, 2014

British Journal of Applied Science & Technology, 4(1): 40-53, 2014

\n
$$
\gamma_m = \frac{\gamma_{1,m}\gamma_{2,m}}{\gamma_{1,m} + \gamma_{2,m} + 1},
$$
\nwhere

\n
$$
\gamma_{2,m} = \left| h_{2,m} \right|^2 \mathcal{E}_m / N_0.
$$
\n(10)

\nBecause $\gamma_{1,m}$ and $\gamma_{2,m}$ are independent, the cumulative distribution function (CDF) of γ_m can be found as

\n
$$
F_{\gamma_m}(\gamma | \Omega_{1,m}, \Omega_{2,m}) = \int_0^\infty P \left(\frac{\gamma_{1,m} \lambda}{\gamma_{1,m} + \gamma_{2,m}} \right) \gamma_{2,m}(\lambda) d\lambda
$$

where

$$
\gamma_{2,m} = |h_{2,m}|^2 \, \varepsilon_m / N_0 \,. \tag{10}
$$

Because $\gamma_{_{1,m}}$ and $\gamma_{_{2,m}}$ are independent, the cumulative distribution function (CDF) of $\,{\gamma}_{_{m}}\,$ can be found as

British Journal of Appled Science & Technology, 4(1): 40-53, 2014
\n
$$
\gamma_{n} = \frac{\gamma_{1,m} \gamma_{2,m}}{\gamma_{1,m} + \gamma_{2,m} + 1},
$$
\n(9)
\n
$$
\gamma_{2,m} = \left| h_{2,m} \right|^2 \mathcal{E}_m / N_0.
$$
\n(10)
\nBecause $\gamma_{1,m} \text{ and } \gamma_{2,m} \text{ are independent, the cumulative distribution function (CDF) of } \gamma_m \text{ can}$
\nbe found as
\n
$$
F_{\gamma_n}(\gamma | \Omega_{1,m}, \Omega_{2,m}) = \int_0^m P \left(\frac{\gamma_{1,n} \lambda}{\gamma_{1,n} + \lambda + 1} \leq \gamma \right) f_{\gamma_{2,n}}(\lambda) d\lambda
$$
\n
$$
= \int_0^T P \left(\gamma_{1,m} \geq \frac{\gamma \lambda + \gamma}{\lambda - \gamma} \right) f_{\gamma_{2,n}}(\lambda) d\lambda + \int_0^a P \left(\gamma_{1,m} \leq \frac{\gamma \lambda + \gamma}{\lambda - \gamma} \right) f_{\gamma_{2,n}}(\lambda) d\lambda
$$
\n(11)
\n
$$
= I_{1,m} + I_{2,m},
$$
\nwhere
\n
$$
I_{1,m} \sqcup \int_0^x P \left(\gamma_{1,m} \geq \frac{\gamma \lambda + \gamma}{\lambda - \gamma} \right) f_{\gamma_{2,n}}(\lambda) d\lambda = \int_0^x f_{\gamma_{2,m}}(\lambda) d\lambda = 1 - e^{-\frac{\gamma}{\Omega_{2,m}}},
$$
\n(12)
\nand
\n
$$
I_{2,m} = \int_0^x P \left(\gamma_{1,m} \leq \frac{\gamma \lambda + \gamma}{\lambda - \gamma} \right) f_{\gamma_{2,m}}(\lambda) d\lambda = \frac{1}{\Omega_{2,m}} \int_0^a \left(1 - e^{-\frac{\gamma \lambda + \gamma}{\Omega_{2,m}} \right) e^{-\frac{\gamma}{\Omega_{2,m}}} d\lambda
$$
\n(13)
\nand
\n
$$
I_{2,m} = \int_0^a P \left(\gamma_{1,m} \leq \frac{\gamma \lambda + \gamma}{\lambda - \gamma} \right) f_{\gamma_m}(\lambda) d\lambda = \frac{1}{\Omega_{2,m}} \int_0^a
$$

where

$$
I_{1,m} \sqcup \int_0^{\gamma} P\left(\gamma_{1,m} \geq \frac{\gamma \lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda) d\lambda = \int_0^{\gamma} f_{\gamma_{2,m}}(\lambda) d\lambda = 1 - e^{-\frac{\gamma}{\Omega_{2,m}}}, \qquad (12)
$$

$$
= \int_{0}^{r} P\left(\gamma_{1,m} \geq \frac{\gamma \lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda) d\lambda + \int_{\gamma}^{\infty} P\left(\gamma_{1,m} \leq \frac{\gamma \lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda) d\lambda \qquad (11)
$$
\n
$$
= I_{1,m} + I_{2,m},
$$
\nwhere\n
$$
I_{1,m} \sqcup \int_{0}^{r} P\left(\gamma_{1,m} \geq \frac{\gamma \lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,m}}(\lambda) d\lambda = \int_{0}^{r} f_{\gamma_{2,m}}(\lambda) d\lambda = 1 - e^{-\frac{\gamma}{\Omega_{2,m}}}, \qquad (12)
$$
\nand\n
$$
I_{2,2} = \int_{\gamma}^{\infty} P\left(\gamma_{1,m} \leq \frac{\gamma \lambda + \gamma}{\lambda - \gamma}\right) f_{\gamma_{2,n}}(\lambda) d\lambda = \frac{1}{\Omega_{2,m}} \int_{\gamma}^{\infty} \left(1 - e^{-\frac{\gamma \lambda + \gamma}{\Omega_{2,m}}}\right) e^{-\frac{\lambda}{\Omega_{2,m}}}\frac{1}{\Omega_{2,m}} d\lambda \qquad (13)
$$
\n
$$
= e^{-\frac{\gamma}{\Omega_{2,m}}} - \frac{1}{\Omega_{2,m}} e^{-\frac{\gamma}{\Omega_{2,m} \Omega_{2,m}}}\int_{0}^{\infty} e^{-\frac{\gamma \lambda \gamma}{\Omega_{2,m} \Omega_{2,m}}}\frac{1}{\Omega_{2,m}} e^{-\frac{\gamma}{\Omega_{2,m} \Omega_{2,m}}}\frac{1}{\Omega_{2,m}} \left(\sqrt{\frac{\gamma^{2} + \gamma}{\Omega_{1,m} \Omega_{2,m}}}\right),
$$
\nwhere $K_{1}(.)$ is a modified Bessel function of the second kind. The third equality was obtained by changing the integration variable from λ to $\lambda + \gamma$. From (12) and (13) the following is obtained\n
$$
F_{\gamma_{m}}(\gamma \mid \Omega_{1,m}, \Omega_{2,m}) = 1 - 2\sqrt{\frac{\gamma^{2} + \gamma}{\Omega_{1,m} \Omega_{2,m}}} e^{-\gamma \left(\frac{\Omega_{1,m} \Omega_{2,m}}{\Omega_{1,m} \Omega_{2,m}}\right)} K_{1}\left(2\sqrt{\frac{\gamma^{2} + \gamma}{\Omega
$$

where $K_1(.)$ is a modified Bessel function of the second kind. The third equality was obtained by changing the integration variable from λ to $\lambda + \gamma$. From (12) and (13) the $\frac{1}{1+\gamma} e^{-\gamma \left(\frac{\Omega_{1,m} + \Omega_{2,m}}{\Omega_{1,m} \Omega_{2,m}}\right)} K_1\left(2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m} \Omega_{2,m}}}\right),$

the second kind. The third equality v

rom λ to $\lambda + \gamma$. From (12) and (13)
 $\frac{\lambda^{1} + \Omega_{2,m}}{\Omega_{2,m}}K_1\left(2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m} \Omega_{2,m}}}\right).$

$$
F_{\gamma_m}(\gamma \mid \Omega_{1,m}, \Omega_{2,m}) = 1 - 2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m}\Omega_{2,m}}} e^{-\gamma \left(\frac{\Omega_{1,m} + \Omega_{2,m}}{\Omega_{1,m}\Omega_{2,m}}\right)} K_1\left(2\sqrt{\frac{\gamma^2 + \gamma}{\Omega_{1,m}\Omega_{2,m}}}\right).
$$
(14)

British Journal of Applied Science & Technology, 4(1): 40-53, 2014

To find the average outage probability for path $m \in [1, M]$, let's compute the expected

value of (14) with respect to $\Omega_{1,m}$ and $\Omega_{2,m}$, i.e. E_{Ω value of (14) with respect to $\Omega_{1,m}$ and $\Omega_{2,m}$, i.e. $E_{\Omega_{1,m}\Omega_{2,m}}\Big|\,F_{_{\gamma_m}}(\gamma\,|\,\Omega_{1,m},\Omega_{2,m})\Big|$ *umal of Applied Science & Technology, 4(1): 40-53, 2014*

ath $m \in [1, M]$, let's compute the expected

, i.e. $E_{\Omega_{1,m}\Omega_{2,m}}\left[F_{\gamma_m}(y \,|\, \Omega_{1,m}, \Omega_{2,m})\right]$, which is
 $\sum_{\Omega_{2,m}} (\Omega_{2,m}) d\Omega_{1,m} d\Omega_{2,m}$. $m \in [1, M]$ (15)

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the average outage probability for path $m \in [1, M]$, let's compute the expected
 $\{ (14)$ with respect to $\Omega_{1,m}$ and $\Omega_{2,m}$, i.e. $E_{\Omega_{1,n},\Omega_{2,m}}\left[F_{\$

$$
P_m(\gamma) = \int_0^\infty \int_0^\infty F_{\gamma_m}(\gamma \, | \, \Omega_{1,m}, \Omega_{2,m}) f_{\Omega_{1,m}}(\Omega_{1,m}) f_{\Omega_{2,m}}(\Omega_{2,m}) d\Omega_{1,m} d\Omega_{2,m}. \quad m \in [1, M] \tag{15}
$$

As mentioned earlier, the method of "selection combining" is used at the receiver. Therefore the average outage probability at the DMS is the product of the outage probabilities for all the paths 1 , i.e.

$$
P(\gamma) = \prod_{m=0}^{M} P_m(\gamma).
$$
 (16)

British Journal of Applied Science & Technology, 4(1): 40-53, 2014

The average outage probability for path $m \in [1, M]$, let's compute the expected

(14) with respect to $\Omega_{1,m}$ and $\Omega_{2,m}$, i.e. $E_{\Omega_{1,n}\Omega_{2,m}}[F_{r_n}(y'|\Omega$ *British Journal of Applied Science & Technology, 4(1): 40-53, 2014*
 the average outage probability for path $m \in [1, M]$, let's compute the expected
 f (14) with respect to $\Omega_{1,m}$ and $\Omega_{2,m}$, i.e. $E_{\Omega_{1,n}\Omega_{2,m}}\Big$ The distributions of $\Omega_{1,m}$ and $\Omega_{2,m}$ are functions of r_m and d_m respectively. In order to obtain useful and informative results, the expected values of these parameters are used (i.e. *British Journal of Applied Science & Technology, 4(1):40-53, 201*
 Entish Journal of Applied Science & Technology, 4(1):40-53, 201
 Exalted of (14) with respect to Ω_{1_m} and Ω_{2_m} , i.e. $E_{\Omega_{1n} \Omega_{2m}}\left[F_{\gamma_n}(Y$ *British Journal of Applied Science & Technology, 4(1): 40-83, 2014*

and fit a verage outage probability for path $m \in [1, M]$, let's compute the expected

of (14) with respect to $\Omega_{1,m}$ and $\Omega_{2,m}$, i.e. $E_{\Omega_{1,n}\Omega_{2,n}}$ British Journal of Applied Science & Technology, 4(1): 40-53, 2014

9ge outage probability for path $m \in [1, M]$, let's compute the expected

in respect to $\Omega_{1,m}$ and $\Omega_{2,m}$, i.e. $E_{\Omega_{1,n}\Omega_{1,m}}\left[F_{\gamma_n}(y \mid \Omega_{1,m}, \Omega_{2,m})\right$ British Journal of Applied Science & Technology, 4(1): 40-53, 2014

iity for path $m \in [1, M]$, let's compute the expected

and $\Omega_{2,m}$, i.e. $E_{\Omega_m,\Omega_{2,m}}\left[F_{\gamma_m}(y \mid \Omega_{1,m},\Omega_{2,m})\right]$, which is
 $\Omega_{\Omega_m}(\Omega_{1_m})(\Omega_{\Omega_m}(\Omega_{2,m})d\$ *British Journal of Applied Science & Technology, 4(1): 40-53, 2014*

To find the average outage probability for path $m \in [1, M]$, let's compute the expected

value of (14) with respect to $\Omega_{1,m}$ and $\Omega_{2,m}$, i.e. F_{Ω and $E[d_m]$ are computed as a function of *m*, which is ordered according to the proximity of the RMSs to the DMS and the number of MSs randomly distributed inside a circle of radius *R* around the DMS. To find the average outage probability for path $m \in [1, M]$, lets compute the expected
value of (14) with respect to $\Omega_{1,m}$ and $\Omega_{2,m}$, i.e. $E_{\Omega_{1,n}1}$, $\Omega_{2,m} \Omega_{2,m} \Omega_{2,m} \Omega_{1,m}$. Which is
given by
given by
 $P_m(x) = \int$

Fig. 4 shows the outage probability $\tilde{P}(\gamma)$ versus the threshold SNR (γ) for $M \in [0,3]$. To verify the results, a simulation is conducted. The result of simulation is presented on the figure with broken lines. The parameters of the simulation are listed in Table 1 (as well as Table 2 in the appendix). These parameters are also used for the evaluation of the formulas.

Fig. 4. Outage probability versus threshold SNR for a system with AGC at the relay

3.2 Simplified Formula

Here, a simpler formula than given in section 3 is derived for the outage probability of the received SNR at the DMS, when the noise in the relays is ignored. While this assumption is not realistic, it is shown that the simpler formula provides results that are quite close to that in section 3. 2 2 ² **Parameter L R µ D** β
 VALUE 500 **M** 100 **M** -10 **DB** 10 **M** 5
 3.2 Simplified Formula

Here, a simpler formula than given in section 3 is derived for the outagenceived SNR at the DMS, when the noise in t **Entish Journal of Applied Science & Technology, 4(1): 40-53, 2014**
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 2. Simplified Formula
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public RS when the noise in the relays is ig **Example 16 Formula** than given in section 3 is derived for the outage probability of the
 Example 16 Formula than given in section 3 is derived for the outage probability of the

simpler formula than given in section 3 Table 1. Parameters of the simulation
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 L R μ D β σ_0 , $\sigma_{1,M}$, $\sigma_{2,M}$
 $\alpha_{1,M}$ $\sigma_{2,M}$
 $\alpha_{2,M}$
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than given in section VALUE 500 M 100 M -10 DB 10 M 5 B
3.2 **Simplified Formula**
Here, a simpler formula than given in section 3 is derived for the outage probability of the
recoived SNR at the DMS, when the noise in the relays is (gnored. Whi the outage probability of the
red. While this assumption is
s that are quite close to that
wing formula is obtained
 $\alpha_m^2 = \varepsilon_m / N_0$, which yields
nown that the CDF of the γ_m
(17)
 $\ln(\hat{\Omega}_m)$ is a normal random
 σ_{2,m 3.2 Simplified Formula

Here, a simpler formula than given in section 3 is derived for the change probability of the

received SNR at the DMS, withen the noise in the relays is givened. While this assumption is

inct resi

From (1), by ignoring the noise in the relays ($z_{1,m} = 0$), the following formula is obtained

$$
\gamma_m = \frac{|h_{1,m}|^2 \alpha_m^2 |h_{2,m}|^2 \varepsilon_0}{N_0}.
$$

2 \mathbf{v} $\frac{0}{2}u$ u \overline{r} cimplicity *m* $\gamma_m = \frac{\alpha_m^2 N_0}{\varepsilon} \gamma_{1,m} \gamma_{2,m}$. For simplicity let's choose $\alpha_m^2 = \varepsilon_m / N_0$, which yields

 $\gamma_m = \gamma_{1,m} \gamma_{2,m}$ (a product of two exponential R.V.s). It can be shown that the CDF of the γ_m is expressed as 2

b) by ignoring the noise in the relays (
$$
z_{1,m} = 0
$$
), the following formula is obtained
\n
$$
I_{1,m} \Big|^2 \alpha_m^2 \Big| h_{2,m} \Big|^2 \mathcal{E}_0
$$
\n
$$
N_0
$$
\n
$$
N_0
$$
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$$
N_0
$$
\n
$$
N_m \Big| \frac{\alpha_m^2 N_0}{\mathcal{E}_m} \gamma_{1,m} \gamma_{2,m}.
$$
 For simplicity let's choose $\alpha_m^2 = \mathcal{E}_m / N_0$, which yields
\n
$$
I_{1,m} \gamma_{2,m}
$$
 (a product of two exponential R.V.s). It can be shown that the CDF of the γ_m and
\n
$$
F_{\gamma_m}(\gamma | \hat{\Omega}_m) = 1 - \frac{2\sqrt{\gamma}}{\hat{\Omega}_m} K_1(\frac{2\sqrt{\gamma}}{\hat{\Omega}_m}) \qquad s > 0, \ \hat{\Omega}_m > 0.
$$
\n
$$
\hat{\Omega}_m = \sqrt{\Omega_{1,m} \Omega_{2,m}}
$$
 is a lognormal random variable, i.e. $\ln(\hat{\Omega}_m)$ is a normal random

is a lognormal random variable, i.e. $\ln(\hat{\Omega}_m)$ is a normal random pressed as²
 $F_{y_m}(\gamma | \hat{\Omega}_m) = 1 - \frac{2\sqrt{\gamma}}{\hat{\Omega}_m} K_1(\frac{2\sqrt{\gamma}}{\hat{\Omega}_m})$ $s > 0$, $\hat{\Omega}_m > 0$, (17)
 $\hat{\Omega}_m = \sqrt{\Omega_{1,m} \Omega_{2,m}}$ is a lognormal random variable, i.e. $\ln(\hat{\Omega}_m)$ is a normal random

ble with $\hat{\mu}_m = (\mu_{1,m}(d_m) + \mu$

$$
P_m(\gamma) = \frac{1}{\sqrt{2\pi}\hat{\sigma}_m \hat{\Omega}_m} \int_0^\infty \left(1 - \frac{2\sqrt{\gamma}}{\hat{\Omega}_m} K_1 \left(\frac{2\sqrt{\gamma}}{\hat{\Omega}_m}\right)\right) \exp\left(-\frac{(\ln(\hat{\Omega}_m) - \hat{\mu}_m)^2}{2\hat{\sigma}_m^2}\right) d\hat{\Omega}_m, \quad m = 1, 2, \dots, M \tag{18}
$$

betwing the noise in the relays $(z_{1,m} = 0)$, the following formula is obtained
 $\frac{h_{2,m}^2|^2}{2_m} \frac{E_0}{E_0}$.
 $\frac{a_m^2 N_u}{\sum_{m}} \gamma_{2,m}$. For simplicity let's choose $\alpha_m^2 = \varepsilon_m / N_0$, which yields

a product of two expone (1), by ignoring the noise in the relays $(z_{i,m} = 0)$, the following formula is obtained
 $\left| \frac{h_{i,m}}{N_0} \right|^2 \frac{\alpha_m^2}{k_{2,m}} \left| \frac{h_{2,m}}{k_0} \right|^2 \frac{c_0}{k_0}$.

fore, $\gamma_m = \frac{\alpha_m^2 N_0}{\varepsilon_m} \gamma_{i,m} \gamma_{2,m}$. For simplicity let' ig the noise in the relays $(z_{1,m} = 0)$, the following formula is obtained
 $\int_{1}^{2} \frac{\mathcal{L}_0}{\mathcal{L}_m}$.
 $\int_{\frac{\mathcal{L}_m}{\mathcal{L}_m}}^2 \frac{\mathcal{L}_m}{\mathcal{L}_{2m}} \gamma_{1,m} \gamma_{2,m}$. For simplicity let's choose $\alpha_m^2 = \mathcal{L}_m / N_0$, which yi *m* (1), by ignoring the noise in the relays $(z_{i,m} = 0)$, the following formula is obtained
 $= \frac{|h_{i,m}|^2 \alpha_m^2 |h_{i,m}|^2 \epsilon_0}{N_0}$.
 metore, $\gamma_m = \frac{\alpha_m^2 N_0}{\epsilon_m} \gamma_{i,m} \gamma_{2,m}$. For simplicity let's choose $\alpha_m^2 = \epsilon_m / N_0$, ic, it is shown that the simpler formula provides results that are quite close to that

by ignoring the noise in the relays $(z_{1:n} = 0)$, the following formula is obtained

by ignoring the noise in the relays $(z_{1:n} = 0)$, or ϵ_m / N_0 , which yields

hat the CDF of the γ_m

(17)

(17)

(17)

(3) is a normal random

2.

, $m = 1, 2, ..., M$,(18)

le integral produced in

from all $M+1$ channels

sesuming that all $M+1$ This equation involves a single integral, to be contrasted with the double integral produced in the calculation of section 3. The outage happens when received SNR from all *M+*1 channels (channel 0 plus *M* relay channels) fall below a specific threshold. Assuming that all *M*+1 channels are independent, the outage is given by (16). where $\Sigma_{\ell_m} = \sqrt{\frac{\lambda^2 I_{1,m}\lambda^2 I_{2,m}}{n}}$. Is a digitormal random variable, i.e. $\ln(\lambda \ell_m)$ is a normal random variable with $\hat{H}_m = (H_{1,m}(d_m) + H_{2,m}(r_m))/2$ and $\hat{\sigma}_m = (\sigma_{1,m} + \sigma_{2,m})/2$.

Using (17), the probability of outage **a** lognormal random variable, i.e. $\ln(\hat{\Omega}_m)$ is a normal random
 $+\mu_{2,m}(r_m)/2$ and $\hat{\sigma}_m = (\sigma_{1,m} + \sigma_{2,m})/2$.

butage for channel *m* is given by
 $\frac{\sqrt{y}}{\lambda_m} K_1(\frac{2\sqrt{y}}{\hat{\Omega}_m}) \exp\left(-\frac{(\ln(\hat{\Omega}_m) - \hat{\mu}_m)^2}{2\hat{\sigma}_m^2}\right) d\hat$ altable with $\mu_m = (\mu_{1,m}(a_m) + \mu_{2,m}(r_m))/2$ and $\sigma_m = (\sigma_{1,m} + \sigma_{2,m})/2$.

Sing (17), the probability of outage for channel m is given by
 $P_m(y) = \frac{1}{\sqrt{2\pi}\hat{\sigma}_m \hat{\Omega}_m} \int_0^x \left(1 - \frac{2\sqrt{y}}{\hat{\Omega}_m} K_1 \frac{2\sqrt{y}}{\hat{\Omega}_m}\right) \exp\left(-\frac{(\ln(\$ variable with $\hat{\mu}_m = (\mu_{1,m}(d_m) + \mu_{2,m}(r_m))/2$ and $\hat{\sigma}_m = (\sigma_{1,m} + \sigma_{2,m})/2$.

Using (17), the probability of outage for channel m is given by
 $P_m(y) = \frac{1}{\sqrt{2\pi}\hat{\sigma}_m}\hat{\Omega}_m \int_0^{\infty} \left(1 - \frac{2\sqrt{y}}{\hat{\Omega}_m} K_1 \frac{2\sqrt{y}}{\hat{\Omega}_m}\right) \$

.

²This can be obtained as follows: $F_{\gamma_m}(\gamma | \Omega_1, \Omega_2) = \Pr{\gamma_{1,m} \gamma_{2,m} \leq \gamma} = \int_0^\infty \Pr{\gamma_{1,m} \leq \frac{\gamma}{\lambda} | \gamma_{2,m} = \lambda\}} f_{\gamma_{2,m}}(\lambda) d\lambda =$

$$
\int_0^{\infty} (1 - e^{-\gamma/\lambda \Omega_{1,m}}) \frac{1}{\Omega_2} e^{-\lambda/\Omega_{2,m}} d\lambda = 1 - \frac{2\sqrt{\gamma}}{\sqrt{\Omega_{1,m} \Omega_{2,m}}} K_1(\frac{2\sqrt{\gamma}}{\sqrt{\Omega_{1,m} \Omega_{2,m}}})
$$

Generalized version of this proof for ² random variable can be found in 10.

Fig. **5** gives this outage probability for the received SNR versus the threshold SNR for *M*=0,1,2,3, To confirm this result, a simulation has been conducted. The result of simulation is presented on the same figure. The parameters that are used here are listed in Table 1. The figure shows the improvement in outage probability as the number of the relays increases. Since the noise at the relays is neglected, the outage probability is slightly reduced, compared with the one given in section 3.

Now suppose that the source and the DMS fail to communicate due to shadow-fade channel or severity of the noise. It would be interesting to determine the number of relays needed to re-establish communication between the source and the destination. This is assumed to be the minimum number of relays required so that the received SNR at the DMS is above a specific threshold $\,\gamma_{\rm \it th}$, when the direct path SNR, $\,\gamma_{\rm 0}$, is below this threshold . Since the M=0,1,2,3, I to comtrim this result, a simulation has been conducted. I he result of simulation
is presented on the same figure. The parameters that are used here are listed in Table 1.
The figure shows the improvement in **Eq. 5** gives this outage probability for the received SNR versus the threshold SNR for Fe9. 2014
 Fig. 5 gives this outage probability for the received SNR versus the threshold SNR for
 M-2.1.7. To confirm this resul *t*, 1 o contim this result, a simulation has been conducted. I he result of simulation the same figure. The parameters that are used here are listed in Table 1.
the shows the improvement in outage probability as the numb are used here are listed in Table 1.
bility as the number of the relays
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Singlets this outage probability for the received SNR versus the threshold SNR for
1.2.3. To confirm this result, a simulation has been condu **Example 3** Fig. 5 gives this outrage probability for the neceived SNR versus the threshold SNR for 5-0.1,2,3, To confirm this result, a simulation has been conducted. The result of simulation species in equality, as in t bose that the source and the DMS fail to communicate due to shadow-fact of y of the noise. It would be interesting to determine the number of relays hommunication between the source and the destination. This is assume num Boxton Journal of Applied Science & Technology 4f1): 40-53, 2014

Fig. 5 gives this outage probability for the received SNR versus the threshold SNR for

MPD,1,2,3, To confirm this result, a simulation has been conducted.

$$
\Pr\{\gamma \ge \gamma_{th} \mid \gamma_0 \le \gamma_{th}\} = \Pr\{\gamma_1 \ge \gamma_{th} \text{ or } \dots \gamma_M \ge \gamma_{th} \mid \gamma_0 \le \gamma_{th}\} = \Pr\{\bigcup_{k=1}^M \gamma_k \ge \gamma_{th} \mid \gamma_0 \le \gamma_{th}\} = 1 - \Pr\{\bigcap_{k=1}^M \gamma_k \le \gamma_{th}\}.
$$

Therefore we get

$$
\Pr\{\gamma \geq \gamma_{th} \mid \gamma_0 \leq \gamma_{th}\} = 1 - \prod_{m=1}^{M} P_m(\gamma),
$$

where $P_m(\gamma)$ is given by (15). Fig. 6 depicts this probability, using the same parameters listed in section 3. It can be seen that by using just one relay (nearest RMS to DMS) the probability of restoring the communication is 93.5%, while this probability is 99.1%. and 99.9%, when two and three relays are employed between the source and the DMS respectively.

Fig. 5. Outage probability versus threshold SNR, when the relays' noise is ignored

Fig. 6. Probability (%) of re-establishing communication between the source and the DMS, versus the number of relays between them

4. CONCLUSIONS

In this paper the outage probability of a proposed multi-user spatial diversity system for a wireless network was studied. The system employs one or more mobiles stations (MS) to relay the signal between the source and the destination mobile station (DMS), in a shadowfading environment. A formula was derived for the outage probability of the received signalto-noise ratio at DMS, when there are *M* intermediate MSs relay the signal between source and DMS. The study was accomplished by finding the probability distribution of the SNR at the DMS, when there are AGCs at the relays. It was shown that the outage probability reduces as the number of the relays increases. Finally, the probability of restoring communication between the source and the DMS as a function of number of relays between them was studied. For future work, we will examine the cases that the received signal at the relay will be regenerated before re-transmission. Also, various channels other than shadowfade will be studied.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

- 1. Stuber GL. Principles of Mobile Communication. Boston, MA: Kluwer; 1996.
- 2. Laneman JN, Wornell GW. Exploiting Distributed Spatial Diversity in Wireless Networks, Proc. of Allerton Conference on Communications, Controls, and Computing, Urbana-Champagne, IL, Oct; 2000.
- 3. Emamian V, Kaveh M. Combating Shadowing Effects for Systems with Transmitter Diversity by Using Collaboration among Mobile Users, Published in Journal of CIEE. November 2002;(4).
- 4. Simon MK. Alouini MS. Digital Communication over Generalized Fading Channels: A Unified Approach to the Performance Analysis, Wiley & Sons, Inc.; 2000.
- 5. Emamian V, Kaveh M. Combating Shadowing Effects for Systems with Transmitter Diversity by Using Collaboration among Mobile Users, Proceedings of the International Symposium on Communications, Nov 13-16, Taiwan; 2001.
- 6. Sendonaris A, Erkip E, Aazhang B. Increasing uplink capacity via user cooperation diversity, Proceedings of IEEE International Symposium on Information Theory. 1998;156.
- 7. Laneman, JN, Wornell GW. Energy-efficient antenna sharing and relaying for wireless networks, IEEE Wireless Communications and Networking Conference. 2000;1:7–12.
- 8. Laneman JN, Wornell GW, Tse DNC. An efficient protocol for realizing cooperative diversity in wireless networks, in Proc. of ISIT, Washington, DC. June 2001;294.
- 9. Grossglauser M, Tse D. Mobility increases the capacity of ad-hoc wireless networks, Proceedings of IEEE INFOCOM. 2001;3:1360-1369.
- 10. Wells WT, Anderson RL, Cell JW. The distribution of the product of two central or non-central chi-square variates, Anals of Mathematical Statistics, Sep. 1962;33(3):1016-1020.
- 11. Hogg RV, Craig AT. Introduction to Mathematical Statistics, 3rd edition, The Macmillan Company; 1970.
- 12. Huang K, Heath R, Andrews J. Space division multiple access with a sum feedback rate constraint, IEEE Tran. on Sig. Proc. 2007;55(7 Part 2):3879–3891.
- 13. Dimic G, Sidiropoulos C. On downlink beamforming with greedy user selection: performance analysis and a simple new algorithm, IEEE Tran. on Sig. Proc. 2005;53(10 Part 1):3857–3868.
- 14. Jindal N. Antenna combining for the MIMO downlink channel, IEEE Transactions on Wireless Communications. 2008;7(10):3834–3844.
- 15. MIMO Broadcast Channels with Finite Rate Feedback, IEEE Tran. on Inform. Theory; 2006.
- 16. Kim J, Kim H, Park C, Lee K. On the performance of multiuser MIMO systems in WCDMA/HSDPA: Beamforming, feedback and user diversity, IEICE Trans. on Commun. 2006;89(8):2161–2169.
- 17. Corless R, Gonnet G, Hare D. On the Lambert W function, Advances in Computational mathematics. 1996;5(1):329–359.
- 18. Yoo T, Jindal N, Goldsmith A. Multi-antenna downlink channels with limited feedback and user selection, IEEE J. on Select. Areas in Commun. 2007;25(7):1478–1491.
- 19. Caire G, Jindal N, Kobayashi M, Ravindran N. Multiuser MIMO achievable rates with downlink training and channel state feedback, Submitted to: IEEE Trans. Inform. Theory; 2007.

APPENDIX

Average Distance to Destination Mobile

In this appendix the average distance of the *m* nearest MS to the DMS is determined, from the source and the DMS. Let *M* mobile stations that are potential relays (RMSs) be distributed uniformly randomly in a unit circle centered at the destination mobile station (DMS). Denote r_m to be the distance between the DMS and the mth nearest RMS to it, and let d_m be the distance between this RMS and the source (**Example 10. Example 10. Example 10. Example 10. Appled Science & Technology.** \star (1): 40-53, 2014
 APPENDIX

In this appendix the average distance of the *m* nearest MS to the DMS is determined, from

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In this appendix the average distance of the m nearest MS to the DMS is determined, from

the source and the DMS. Let M mobile statio *r* Technology, 4(1): 40-53, 2014

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the source and the DMS. Let M mo EXERCT DRIVER INTO A SURFACT UNIT AND STRUCTURE THE MENTAL CONDUCT TH **EXECUTE:**
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tance to Destination Mobile

the mearest MS to the DMS is determined, from

the DMS. Let M mobile stations that are potential relay (RMSs) be

formly randomly in a unit circle centered at the

Let's begin by determining the probability density function (PDF) of r_m , $f_{r_m}(r)$. We have is located inside the annular [r - Δr , r], M-m RMSs are located inside the annular [r,1]}. *may.* Define n_m to be the distance between the bind and the interaction of the the distance of d_m be the distance between this RMS and the source (
 a 3). r_i are then ordered as $r_i < r_2 < \ldots < r_m < \ldots < r_M$. The averag *m* by determining the probability density function (PDF) of r_m , $f_{r_m}(r)$. We have
 $= Pr\{r - r \le r_m \le r\} = Pr\{m-1 \text{ RMSs are located inside the disk } [0, r-4r], \text{ one RMS} \}$

inside the annular $[r-4r, r]$, $M-m$ RMSs are located inside the annular $[r, 1]\}$. $r_1 < r_2 < < r_m < < r_M$. The average distances $\overline{r}_m = E[r_m]$

MS are derived.

probability density function (PDF) of r_m , $f_{r_m}(r)$. We have
 $r_3 = Pr(rm-1 \text{ RMS are located inside the disk } [0, r-4d]$, one RMS
 $r_2 = Pr(rm-1 \text{ RMS are located inside the annular } [r, 1]).$
 $b^2 = a^2$, dered as $r_1 < r_2 < < r_m < < r_M$. The average distances $\overline{r}_m = E[r_m]$

the mth RMS are derived.

then th RMS are derived.

mining the probability density function (PDF) of r_m , $f_{r_o}(r)$. We have
 $r \le r_m \le r$) =Pr{m-1 RM are then ordered as $r_1 < r_2 < \ldots < r_m < \ldots < r_n$. The average distances $\overline{r}_m = E[f_{m}]$
 $E[f_{m}]$ for the m^{to} RMS are derived.

(in by determining the probability density function (PDF) of r_m , f_{r_m} (*r*). We have
 $r = \Pr\$ *r*, are then ordered as $r_i < r_i < \ldots < r_m < \ldots < r_N$. The average distances $\overline{r}_n = E[r_n]$
 $= F_i[d_n]$ for the m^{on} RMS are derived.
 rr $r = Pr_1 r - r \le r_n < r_i > r_i > r_i = Pr(m + RMSs$ are located inside the disk $[0,r,4t]$, one RMS
 d inside ben this RMS and the source (
 $\frac{1}{r_1} < r_2 < \dots < r_m < \dots < r_M$. The average distances $\overline{r_m} = E[r_m]$

RMS are derived.
 $\frac{1}{r_m} = F[r_m]$

RMS are derived.

The probability density function (PDF) of r_m , f_m (r). We have
 $r_$ the distance between this RMS and the source (

re then ordered as $r_i < r_2 < ... < r_m < ... < r_M$. The average distances $\overline{r}_m = E[r_m]$
 $E[d_m]$ for the m³⁶ RMS are derived.

by determining the probability density function (PDF) of $\{r - r \le r_m \le r\}$ = Pr(m-1 RMSs are located inside the disk [0,r- λr], one RMS

the annular $[r \cdot \lambda r, \eta, M \cdot m$ RMSs are located inside the annular [r,1]).
 $\int r \{a \le r_i \le b\} = b^2 - a^2$, it follows that
 $\binom{M}{m} (r - r)^{2(m-1)} (r$

$$
f_{r_m}(r)\perp r=m\binom{M}{m}(r-r)^{2(m-1)}(r^2-(r-r)^2)(1-r^2)^{M-m}.
$$

Simplifying this equation we get 3

ed inside the annular
$$
[r_1r_2r, r]
$$
, *M-m* RMSs are located inside the annular $[r, 1]$.
\ng that $Pr\{a \le r_i \le b\} = b^2 - a^2$, it follows that
\n
$$
|r = m \binom{M}{m} (r - r)^{2(m-1)} (r^2 - (r - r)^2)(1 - r^2)^{M-m}.
$$
\n
$$
f_m(r) = 2m \binom{M}{m} r^{2m-1} (1 - r^2)^{M-m}.
$$
\n(19)
\nbected value of r_m is then given by
\n
$$
\overline{r}_m = \int_0^1 r \, f_m(r) dr = \frac{\Gamma(M+1) \Gamma(m+\frac{1}{2})}{\Gamma(M+\frac{3}{2}) \Gamma(m)}.
$$
\n(20)
\n(1) is the gamma function. When the neighborhood cell has a radius R, the average
\ne of m^m nearest RMS to the DNS scales to be
\n
$$
\overline{r}_m = R \frac{\Gamma(M+1) \Gamma(m+\frac{1}{2})}{\Gamma(M+\frac{3}{2}) \Gamma(m)}.
$$
\n(21)
\nWe calculate the expected value of the distance between the source and RMS_m (21)
\nWe proof can be found in 11, pp 147-150.

The expected value of r_m^+ is then given by

$$
\overline{r}_{m} = \int_{0}^{1} r \, f_{r_{m}}(r) dr = \frac{\Gamma(M+1)\Gamma(m+\frac{1}{2})}{\Gamma(M+\frac{3}{2})\Gamma(m)}
$$
(20)

where $\Gamma(.)$ is the gamma function. When the neighborhood cell has a radius *R*, the average distance of m^{th} nearest RMS to the DMS scales to be

$$
\overline{r}_m = R \frac{\Gamma(M+1)\Gamma(m+\frac{1}{2})}{\Gamma(M+\frac{3}{2})\Gamma(m)}.
$$
\n(21)

Let's now calculate the expected value of the distance between the source and RMS*^m* (

³ Alternative proof can be found in 11, pp 147-150.

British Journal of Applied Science & Technology, 4(1): 40-53, 2014

Fig. 3) We have $d_m^2 = L^2 + r_m^2 - 2r_m L \cos(\theta)$, where *L* is the distance between source and

the DMS. θ is uniformly distributed over $[-\pi, \pi]$ and indep British Journal of Applied Science & Technology, 4(1): 40-53, 2014

Fig. 3) We have $d_m^2 = L^2 + r_m^2 - 2r_m L \cos(\theta)$, where L is the distance between source and

the DMS. θ is uniformly distributed over $[-\pi, \pi]$ and indepen the DMS. θ is uniformly distributed over $[-\pi, \pi]$ and independent of r_m . Also we have British Journal of Applied Science & Technology, 4(1): 40-53, 201

ve $d_m^2 = L^2 + r_m^2 - 2r_m L \cos(\theta)$, where L is the distance between source and

uniformly distributed over $[-\pi, \pi]$ and independent of r_m . Also we have
 \frac British Journal of Applied Science & Technology, 4(1): 40-53, 2014

= $L^2 + r_m^2 - 2r_m L \cos(\theta)$, where L is the distance between source and

ly distributed over $[-\pi, \pi]$ and independent of r_m . Also we have
 $\frac{L \cos(\theta)}{\frac{2}{$ British Journal of Applied Science & Technology, 4(1): 40-53, 2014

2 change $d_m^2 = L^2 + r_m^2 - 2r_m L \cos(\theta)$, where L is the distance between source and
 θ is uniformly distributed over $[-\pi, \pi]$ and independent of r_m . Al *British Journal of Applied Science & Technology, 4(1): 40-53, 2014*
 m $d_m^2 = L^2 + r_m^2 - 2r_m L \cos(\theta)$, where L is the distance between source and

uniformly distributed over $[-\pi, \pi]$ and independent of r_m . Also we have
 British Journal of Applied Science & Technology, 4(1): 40-53, 2014
 Fig. 3) We have $d_m^2 = L^2 + r_m^2 - 2r_m L \cos(\theta)$, where L is the distance between source and
 the DMS. θ *is uniformly distributed over* $[-\pi, \pi]$ and i *British Journal of Applied Science & Technology, 4(1): 40-53, 2014*
 $= L^2 + r_m^2 - 2r_m L \cos(\theta)$, where L is the distance between source and

mly distributed over $[-\pi, \pi]$ and independent of r_m . Also we have
 $\frac{L \cos(\theta)}{L^$ *British Journal of Applied Science & Technology, 4(1): 40-53, 2014*
 a $d_m^2 = L^2 + r_m^2 - 2r_m L \cos(\theta)$, where L is the distance between source and
 niformly distributed over $[-\pi, \pi]$ and independent of r_m . Also we have British Journal of Applied Science & Technology, 4(1): 40-53, 2014

3) We have $d_m^2 = L^2 + r_m^2 - 2r_m L \cos(\theta)$, where L is the distance between source and

DMS. θ is uniformly distributed over $[-\pi, \pi]$ and independent of British Journal of Applied Science & Technology, 4(1): 40-53, 2014
 $E = L^2 + r_m^2 - 2r_m L \cos(\theta)$, where L is the distance between source and

lily distributed over $[-\pi, \pi]$ and independent of r_m . Also we have
 $\frac{L \cos(\theta)}{2}$

$$
d_m = L\sqrt{1 + \frac{r_m^2 - 2r_m L \cos(\theta)}{L^2}} \approx L\left(1 + \frac{r_m^2 - 2r_m L \cos(\theta)}{2L^2}\right)
$$

expected value of *d^m* is given by

British Journal of Applied Science & Technology, 4(1): 40-53, 2014\n\nFig. 3) We have
$$
d_m^2 = L^2 + r_m^2 - 2r_m L \cos(\theta)
$$
, where *L* is the distance between source and the DMS. θ is uniformly distributed over $[-\pi, \pi]$ and independent of r_m . Also we have\n
$$
d_m = L \sqrt{1 + \frac{r_m^2 - 2r_m L \cos(\theta)}{L^2}} \approx L \left(1 + \frac{r_m^2 - 2r_m L \cos(\theta)}{2L^2}\right)
$$
\n\nWe calculate $E[r_m^2]$ by using (19), which yields $E[r_m^2] = \frac{m}{M+1}R^2$. Knowing that $E[\theta] = 0$, the expected value of d_m is given by\n
$$
\overline{d}_m = L \left(1 + \frac{m}{2(M+1)} \frac{R^2}{L^2}\right).
$$
\n\nTable 2 presents values for \overline{r}_m and \overline{d}_m , when $L = 500$ m, $R = 250$ m and $M = 9$, for $m = 1, 2, 3, 4$.\n\nTable 2. Average distance of the m^{th} nearest MS to the DMS, from the source and the DMS.

Table 2 presents values for \overline{r}_m and d_m , when L=500m, R=250m and M=9, for m=1,2,3,4.

Table 2. Average distance of the *m* **th nearest MS to the DMS, from the source and the DMS**

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