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A Mathematical Modeling of School Feeding Programme in the Asem – Kumasi Cluster of Schools in Ashanti Region of Ghana

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Abstract

This paper sought to study and examine the need to investigate with quantitative data whether the objectives of higher enrolment has been met with the introduction of School Feeding Programme in the Ashanti region of Ghana. Time series analysis was used to analyze the data obtained. A best fit model was found by using the Box Jenkin's approach to time series analysis. Time series and forecasting has been used to model and forecast the enrolment of pupils of schools which were currently enrolled on the Ghana School Feeding Programme with Asem Cluster of Schools as a case study. The forecasted number of pupils who will be enrolled were obtained by using ARIMA (3, 1, 3) model. The AIC for the best selected model was calculated as 1236.17. The study concluded that the school feeding in the region should be continued to attract more pupils.

Keywords: Autocorrelation Function (ACF), Partial Autocorrelation Function (PACF), Mean Average (MA), Autoregressive (AR), Autoregressive Integrated Moving Average (ARIMA), Akaike's Information Criteria (AIC), Schwarz's Bayesian Information Criteria (BIC).

1 Introduction

It is of imperative concern to note that according to the United Nations World Food Programme, 66 million primary school going age children go hungry every day, with 23 million hungry children in Africa alone [1]. A large number of such children are concentrated in few countries in which lots of them are girls. World leaders have pledged to curb the incidence of hunger in their countries but the developing countries need to put in more efforts. Governments and development organizations put by appreciable amount of money to provide free school meals to needy children.

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In 2008, the World Food Programme (WFP) operated school feeding programmes in 68 poor countries, including most of Africa [2]. Governments have a part to play in the feeding programme [3]. A national meal programme can increase enrollment [4].

School feeding programme is explained as directed social safety cover that provide both educational and health benefits to the most vulnerable children. Ghana School Feeding Programme (GSFP) is a pilot project to provide food to children at school. Governments are entreated to own the school meals [5].

This paper adds to the literature by evidently studying the trend of school feeding programme and establishing beyond every reasonable doubt that it has the potential of strengthening school attendance and enrollment.

1.1 Objectives of the Paper

The objectives of this paper are:

1. To observe the pattern of school pupils enrollment of Asem cluster of schools in the Ashanti Region of Ghana from 2005 to 2012.
2. To model the school pupils enrolment of Asem cluster of schools in the Ashanti Region of Ghana.
3. To forecast the selected model to check whether the school pupils enrolment in the region will be increased on not.

1.2 Significance of the Reported Research

The significance of the contribution in this paper can be summarized as follows:

i) This paper provides a method for assessing the effectiveness of school feeding programme.
ii) This paper contributes to the research information on school feeding programme in the country, so that it can help in further work in the area of research to investigate the effects of the programme on Ghanaians.
iii) The paper attempts to present both application and theory at a level accessible to a wide variety of students, practitioners and researchers.

1.3 Problem Statement

There are a variety of challenges that rear their heads up in the creation and implementation of school feeding programmes. In order to have a successful programme, countries must determine if school feeding is the most effective programme that can be offered to target the neediest children in countries and this paper would like to survey that.

1.4 Limitations of the Paper

i) The duration spanned the time from when the scheme was introduced to the stated date (2005 - 2012).
ii) The scope of research was limited to the Asem cluster of schools in the Ashanti Region of Ghana.
iii) No questionnaires were issued.

From the study of the open literature no research work has been so far reported on the model of school feeding in the Ashanti Region of Ghana. This paper therefore seeks to fill the research gap by studying the effectiveness of school feeding programme in the Ashanti Region of Ghana using time series analysis.

The remaining part of this paper is organized as follows. Section 1.5 surveys the research work relating to the subject of the paper. Section 2 discusses the data used, how they were obtained as well as the methods applied on them to obtain the needed results. Section 3 deals with data analysis, modeling and forecasting. Section 4 contains the summary and findings, conclusions and the recommendations of the paper.

2 Methodologies

A time series is an ordered sequence of values of a variable at equally spaced time intervals. A time series may be measured continuously or discretely. Time series is used for two main purposes,

i. Obtain an understanding of the underlying forces and structure that produced the observed data.
ii. Fit a model and proceed to forecasting, monitoring, or even feedback and feed-forward.

The Box-Jenkins (ARIMA) [6] method differences the series to stationarity and then combines the moving average with autoregressive parameters to yield a comprehensive model amendable to forecasting. This method for analyzing stationary univariate time series data was developed by, George E. P. Box and Gwilym M. Jenkins in 1972. The model developed serves not only to explain the underlying process generating the series, but also as a basis for forecasting.

The univariate version of this methodology is a self- projecting time series forecasting method. The underlying goal is to find an appropriate formula so that the residuals are as small as possible and exhibit no pattern. The model building process involves a few steps, repeated as necessary, to end up with a specific formula that replicates the patterns in the series as closely as possible and also produces accurate forecasts.

The basis of the Box-Jenkins approach to modeling time series is summarized below and consists of three phases: identification, estimation and testing, and application [7].

2.1 Identification Stage

Transform data to stabilize variance.

2.1.1 Difference data to obtain stationary series

For non-seasonal data, first differencing is usually sufficient to attain stationarity. The first-order difference is denoted by equation (1).

$$\nabla Y_t = Y_t - Y_{t-1} \tag{1}$$

2.1.2 Model selection

Examine data, Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) to identify potential models. The autocorrelation function (ACF), $\rho_{t,s}$, is given by the formula in equation (2)

$$\rho_{t,s} = Corr(Y_t, Y_s) = \frac{Cov(Y_t, Y_s)}{\sqrt{Var(Y_t)Var(Y_s)}} \quad \forall\ t, s \in \{0, \pm1, \pm2, \pm3, \dots\} \tag{2}$$

where $Cov(Y_t, Y_s) = E[(Y_t - \mu_t)(Y_s - \mu_t)] = E(Y_t, Y_s) - \mu_t \mu_s$

The autocorrelation coefficient estimated from sample observations at lag *k* is given by equation (3).

$$\gamma_k = \frac{\sum_{t=k+1}^{n}(Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{n}(Y_t - \bar{Y})^2} \quad \text{(Spyros, 1998)} \tag{3}$$

2.2 Testing Stage

2.2.1 Augmented Dickey-Fuller (ADF) test

The hypotheses H_0: Y_t is non – Stationary and H_1: Y_t is Stationary can be tested in the regression equation (4).

$$\Delta Y_t = \beta_0 + \alpha t + \beta_1 Y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta Y_{t-i} + \varepsilon_t \tag{4}$$

Accept H_0 if $P - value > 0.05$, else accept H_1.

2.2.2 Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test

An alternative approach to the ADF test is the KPSS test. A hypotheses of H_0: Y_t is level or trend stationary is tested against H_1: Y_t is non- stationary in the regression equation in (5).

$Y_t = \alpha_t + \beta t + \mu_t$, where a random walk, $\alpha_t = \alpha_{t-1} + \varepsilon_t$ is allowed. (5)

Accept H_0 if $P - value > 0.05$, else accept H_1.

2.3 Select Best Model using Suitable Criterion

2.3.1 The Akaike's Information Criteria (AIC)

The AIC is equal to twice the number of parameters in the model minus twice the logarithm of the likelihood function. Mathematically, AIC is calculated by equation (6).

$$AIC(p,q) = 2k - 2\log(\text{Maximum Likelihood}) \quad (6)$$

where $k = p + q + 1$ if the model contains an intercept or constant term and $k = p + q$ otherwise.

Given two or more competing models, the one with the smaller AIC value will be deemed more appropriate [7].

2.3.2 The Schwartz's Bayesian Information Criteria (BIC)

Like the AIC, the BIC is an order selection criterion for ARIMA models. It is defined mathematically by equation (7).

$$BIC(p,q) = k \times \log(n) - 2 \times \log(\text{Max. Likelihood}) \quad (7)$$

2.4 Diagnostics

2.4.1 Testing the model for adequacy (portmanteau test)

After identifying an appropriate model for a time series data, it is very important to check that the model is adequate. [8] provided a modified portmanteau test statistic for checking the randomness of the error terms. Their statistic is given by equation (8),

$$Q^* = n(n+2) \times \sum_{k=1}^{h} \left(\frac{r_k^{\,2}}{n-k} \right) \quad (8)$$

which is approximately distributed as a χ^2 with $h - p - q$ degrees of freedom, where n is the length of the time series, h is the first h autocorrelations being checked, p is the order of the AR process and q is the order of the MA process and r is the estimated autocorrelation coefficient of the k^{th} residual term. If the calculated value of Q^* is greater than χ^2 for $h - p - q$ degrees of freedom, then the model is considered inadequate and the model is adequate if Q^* calculated is less than χ^2 for $h-p-q$ degrees of freedom. If the model is tested inadequate, then the forecaster should select an alternative model and test for the adequate of the model [7].

3 Data Analysis and Results

This section deals with data analysis, modeling and forecasting.

3.1 Time Plot of Termly Enrolment Figure for Each Basic School Class in the Ashanti Region

Fig. 1 shows the termly time series plot of enrolment figure for each basic school class from 2005 to 2012.

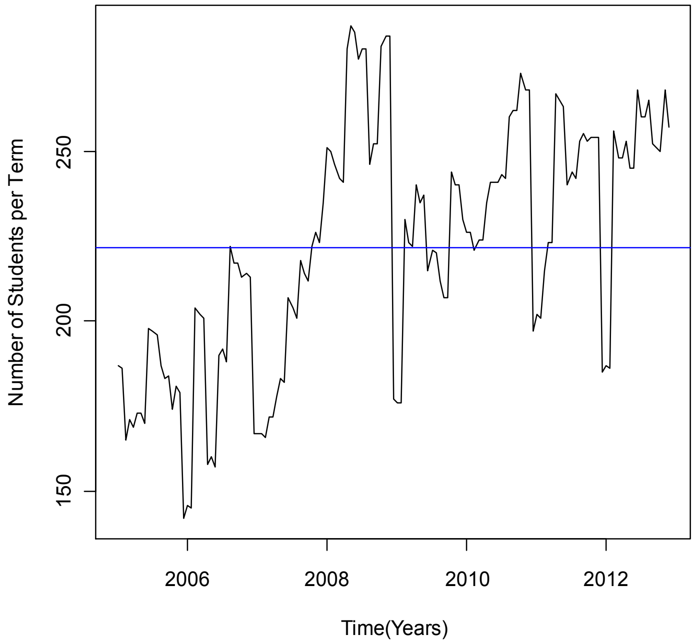

Fig. 1. Termly time series plot of enrolment figure for each basic school class from 2005 to 2012

The enrolment plot fluctuates throughout the period as shown in Fig. 1. A careful observation of the enrolment plot from 2005 to 2012 indicates that there was an increase in enrolment. From the year 2005, there was a gradual increase in enrolment till 2006 when there was a sharp decline. The enrolment increased again from 2007 to 2009 but there was a sharp decrease in 2009. The plot is non-stationary due to the trend component present.

In general, the trend in school enrolment in the Ashanti Region of Ghana seams to be irregular. The annual time plot in Fig. 1 does not exhibit seasonal variation. The mean line is indicated blue. Most of the data points are a little bit far apart from the mean. This indicates that there is a clear case of non-stationarity in the mean.

3.1.1 Checking for Stationarity Using the autocorrelation function (ACF) and partial autocorrelation function (*PACF)*

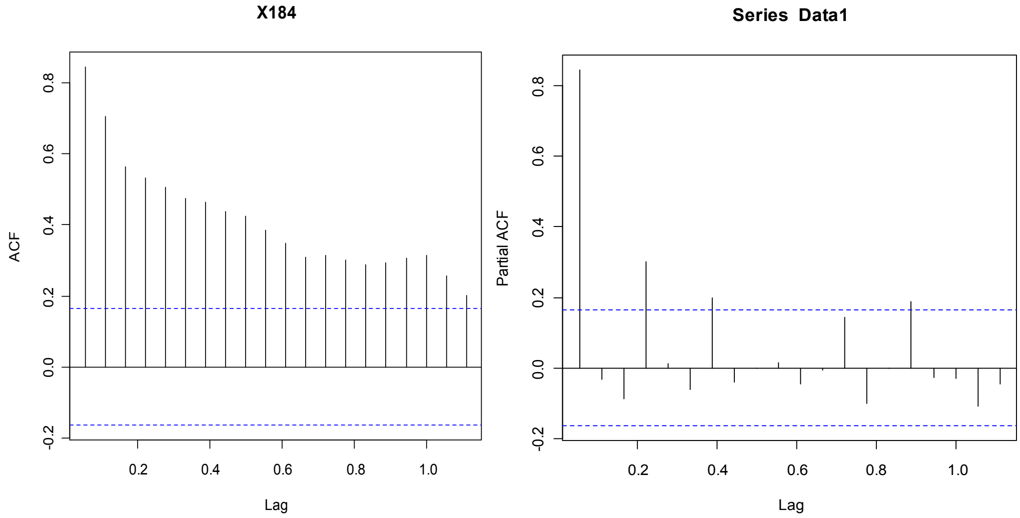

Fig. 2 (i) **Fig. 2 (ii)**

Fig. 2. The autocorrelation function and partial auto-correlation function of school enrolment

The Correlogram, shown in Fig. 2 (i), describes the correlation between the enrolments in different years as a function of time. The autocorrelation function is decreasing gradually and that means there is a non-stationary in the enrolment figures. Fig. 2 (ii) exhibits the partial autocorrelation function (PACF) of the enrolment data. At lag zero the PACF is almost unity (1) which confirms that the enrolment time series is non-stationary.

Table 1. Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests

Augmented Dickey-Fuller (ADF)Test		
Dickey-Fuller	**Lag order**	**P-value**
-3.4155	5	0.05482
Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test		
KPSS level	**Lag parameter**	**P-value**
2.4357	2	0.01

From Table 1 the p-value for ADF test was greater than 0.05 and that of KPSS was less than 0.05 using a 5% significant level. The two tests confirmed that there was non-stationarity in the original school enrolment data which needs to be differenced to achieve mean stationarity.

3.2 First-Order Differencing of Termly Enrolment Figure for each Basic School Class in the Ashanti Region of Ghana from 2005 to 2012

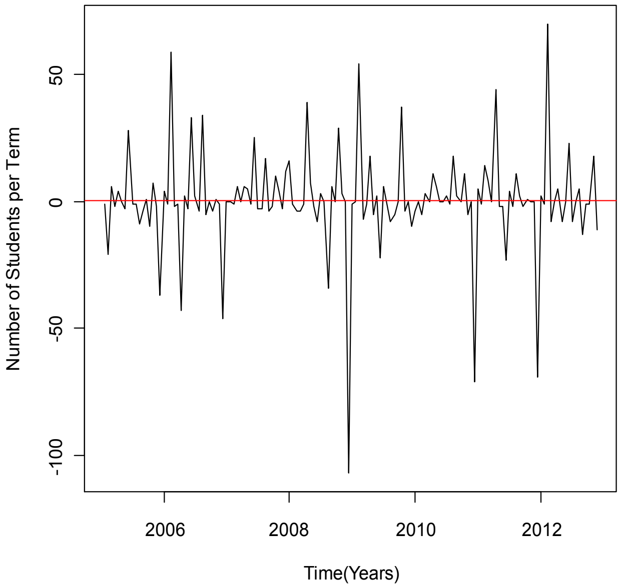

Fig. 3. First-order differencing of termly enrolment figure for each basic school class in the Ashanti Region of Ghana

After first differencing, we saw that the observations revert to the mean value as shown in Fig. 3. The red line here depicts the mean. Thus the observations in the plot of first differencing move irregularly but revert to their mean value with an approximately constant variability. It follows from Fig. 3 that there is stationarity. In general terms, there was no seasonal behavior in the time plot, and the school enrolment data now looks to be approximately stable for further investigations.

3.2.1 Objective test for stationarity for the first-order differenced series

From Table 2, the p-value for ADF test was less than 0.05 and that of KPSS was greater than 0.05. It follows that the school enrolment data now looks to be approximately stationary in the mean for further investigations.

Table 2. Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests

Augmented Dickey-Fuller (ADF)Test		
Dickey-Fuller	**Lag order**	**P-value**
-7.3238	5	0.01
Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test		
KPSS level	**Lag parameter**	**P-value**
0.0151	2	0.1

3.3 Selecting Competing Models Using ACF and PACF of the First-Order Differencing of School Enrolment

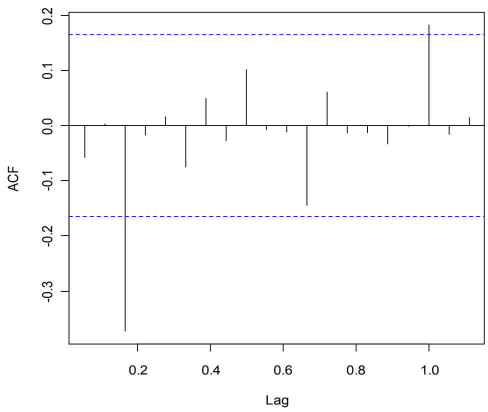

Fig. 4 (i)

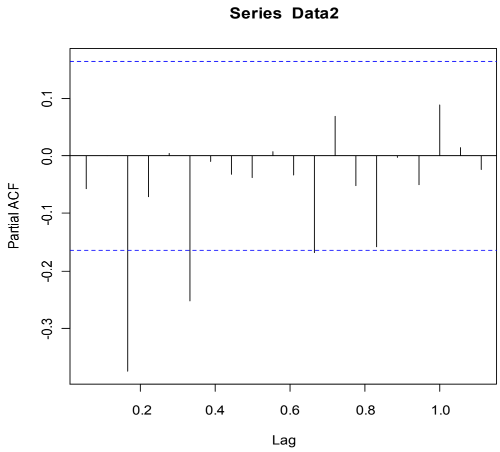

Fig. 4 (ii)

Fig. 4. ACF and PACF of first-order differencing of school enrolment in the Ashanti Region of Ghana

From Fig. 4 (i), the PACF was tailing off and the ACF was cutting off at spikes 3 suggested an MA (3) model. Fig. 4 (ii) shows the sample Partial Auto-Correlation Function (PACF) of the first order differencing of the school feeding programme in the Ashanti Region of Ghana at different lags. The ACF was tailing off and the PACF was cutting off at spikes 3 and 6 suggested an AR (3) model or AR (6) model.

As a preliminary analysis, we will fit both models. It follows that, in both the ACF and the PACF of the first order differencing of the enrolment data, the following models were suggested:

ARIMA (0, 1, 3),
ARIMA (3, 1, 0) and
ARIMA (3, 1, 3)

3.4 Estimation of Tentative Models

3.4.1 Parameter estimate and diagnostics of ARIMA (0, 1, 3) model

Table 3. Parameter estimate for ARIMA (0, 1, 3) with non-zero mean

Coefficient	Estimate	Standard error	/t-value/
ma1	-1.109	0.1317	8.4207
ma2	0.0342	0.0956	0.3577
ma3	0.0748	0.1337	0.5595
AIC	**AICc**	**BIC**	**Constant**
1252.44	1252.88	1267.18	0.0033

The coefficients of the estimated MA (3) parameters were not within the invertibility condition bounds since the absolute t-values of ma2 and ma3 were less than 2 as shown in Table 3. From Table 3, the estimated model for ARMA (0, 3) was given by equation (9):

$$Y_t = -1.109\omega_{t-1} + 0.0342\omega_{t-2} + 0.0748\omega_{t-3} + 0.0033 + \omega_t \tag{9}$$

Table 4. Box-Ljung test of ARIMA (0, 1, 3) with non- zero mean

Box - Ljung Test		
X squared	**d f**	**p-value**
33.2067	20	0.0320

Results from Table 4 showed that the model's residuals were significant with Ljung Box test statistic of 33.2067 and a p-value of 0.0320. Hence the model was not adequate for forecasting since its p-value was less than 0.05.

3.4.2 Parameter estimate and diagnostics of ARIMA (3, 1, 0) model

Table 5. Parameter estimate for ARIMA (3, 1, 0) with non-zero mean

Coefficient	**Estimate**	**Standard error**	**/t-value/**
ar1	-0.6174	0.0773	7.9871
ar2	-0.3224	0.0891	4.3569
ar3	-0.3882	0.0770	5.0416
AIC	**AICc**	**BIC**	**Constant**
1282.96	1283.40	1297.70	0.0006

The coefficients of the estimated AR(3) parameters were within the causality condition bounds since the absolute t-values were all greater than 2 as shown in Table 5. The estimated model for ARMA (3) was given by equation (10):

$$Y_t = -0.6174\,Y_{t-1} - 0.3224\,Y_{t-2} - 0.3882\,Y_{t-3} + 0.0006 + e_t \tag{10}$$

Table 6. Box-Ljung test of ARIMA (2, 1, 0) with non- zero mean

Box - Ljung Test		
X squared	**d f**	**p-value**
35.8667	20	0.0160

Results from Table 6 shows that the model's residuals were significant with Ljung Box test statistic of 35.8667 and a p-value of 0.0160. Hence the model was not adequate for forecasting since its p-value was less than 0.05.

3.4.3 Parameter estimates and diagnostics of ARIMA (3, 1, 3) model

Table 7. Parameter estimate for ARIMA (3, 1, 3) with non-zero mean

Coefficient	Estimate	Standard error	/t-value/
ar1	0.2564	0.1857	1.3807
ar2	0.2071	0.1835	1.1286
ar3	-0.3343	0.0904	3.6637
ma1	-1.3812	0.2012	6.8648
ma2	0.1479	0.2848	0.5193
ma3	0.2333	0.1900	1.2280
AIC	AICc	BIC	Constant
1236.17	1237.26	1259.76	0.0023

The coefficient of the estimated ARMA (3, 3) parameters was not within the causality condition bounds as shown in Table 7. The estimated ARMA (3, 3) model was given by equation (11).

$$Y_t = 0.2564Y_{t-1} + 0.2071Y_{t-2} - 0.3343Y_{t-3} - 1.3812\omega_{t-1} + 0.1479\omega_{t-2} + 0.2333\omega_{t-3} + 0.0023 \quad (11)$$

Table 8. Box-Ljung test and forecasts from ARIMA (3, 1, 3) with non- zero mean

Box - Ljung Test		
X squared	**d f**	**p-value**
17.6008	20	0.6137

Results from Table 8 showed that the model's residuals were non-significant with Ljung Box test statistic of 17.6008 and a p-value of 0.6137. Hence the model was adequate for forecasting since its p-value was greater than 0.05.

3.5 Model Selection and Forecasting Using AIC and p-Value of Box-Ljung Test

The standardized residual plots of all the models had constant mean and few outliers. There was significance in the autocorrelation functions of the residuals of all the models. The residuals appeared to be normally distributed in all the models.

Table 9. Statistics of ARIMA models

Model	AIC	AICc	BIC	P-Value
ARIMA(3,1,0)	1282.96	1283.40	1297.70	0.0160
ARIMA(0,1,3)	1252.44	1252.88	1267.18	0.0320
ARIMA(3,1,3)	1236.17	1237.26	1259.76	0.6137

From Table 9 the ARIMA (3, 1, 3) model was chosen as the best model for forecasting since it had the minimum AIC_c and AIC values and also its p-value was greater than 0.5.

3.5.1 Forecasting from ARIMA (3, 1, 3)

Based on the observed data, predictions for the next year, 2013 was made. Fig. 5 showed the prediction points and the confidence interval of the forecasts.

The model was used to forecast one year ahead and showed that the school enrolment in the Ashanti Region of Ghana will be slightly increased from 2012 to 2013 as shown in the blue line of Fig. 5 as well as Table 10, its forecast points.

Table 10. Forecast for the 2012/2013 academic year

Basic	Terms	Forecast
1	1	250
	2	256
	3	261
2	1	260
	2	262
	3	264
3	1	264
	2	265
	3	266
4	1	266
	2	267
	3	268
5	1	268
	2	269
	3	269
6	1	270
	2	271
	3	271

From Fig. 5, the yellow region depicts the 95% confidence interval, the red region is the 85% confidence interval and the blue line is the forecasting points.

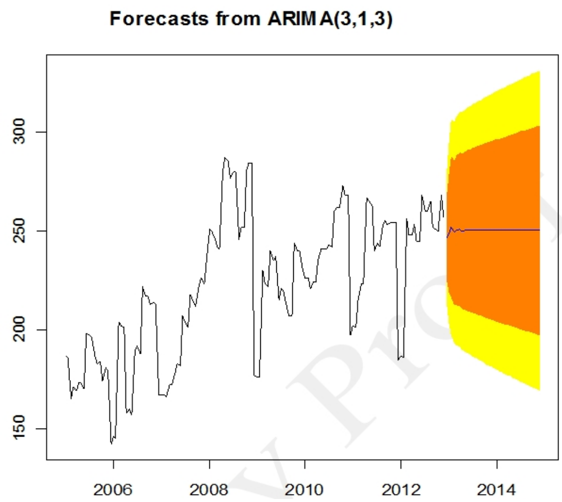

Fig. 5. Forecasts from ARIMA (3, 1, 3) with non-zero mean

4 Conclusion, Findings and Recommendation

This section contains the conclusions, summary and findings and the recommendations of the paper.

4.1 Conclusion

Results from Table 9 showed that ARIMA (3, 1, 3) had the lowest AIC value of 1236.17. It was the best model so far as the AIC was concerned. It follows from our results that the required equation for the best selected model was

$$Y_t = 0.2564\, Y_{t-1} + 0.2071\, Y_{t-2} - 0.3343\, Y_{t-3} - 1.3812\, \omega_{t-1} + 0.1479\, \omega_{t-2} + 0.2333\, \omega_{t-3} + 0.0023 \cdot$$

In conclusion, the research study reported in this monograph has found that school feeding data in the Ashanti Region of Ghana could best be modeled with ARIMA (3, 1, 3). The study again found out that the enrolment for basic schools in the Ashanti Region is expected to increase so the government should continue the feeding programme. Quality school meal can improve test scores [9]. Consistent school feeding will curb stunted growth [10].

4.2 Findings

Results from this paper shows that continuous use of feeding programme is having a significant impact on the rate of school enrolment in the Ashanti region of Ghana.

From definitions, the school enrolment data in the Ashanti Region of Ghana from 2005 to 2012 depicts a *Stochastic Time Series* and its model is *linear*.

The results also showed from Table 9 that the higher the AIC the smaller its p-value of Box-Ljung test and the smaller the AIC the bigger its p-value of Box-Ljung test.

4.3 Recommendations

The following recommendations were made after careful analyses of the data. The school feeding programme should be continued. Based on the analysis, enrolment for basic schools has risen over the seven year period.

The teachers must be involved in the process of feeding the pupils, from the purchase of foodstuffs through food preparations to food services. This is because they are directly responsible for the nutritional needs of the pupils during class hours.

The food on the School Feeding menu should be examined to make the food more nutritious for the mental advancements of the pupils.

Competing Interests

Authors have declared that no competing interests exist.

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