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On the Existence and Nonexistence of Positive Solutions for Singular Quasilinear Elliptic Equations with Gradient Terms

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Abstract

In this paper, we investigate the positive solutions of quasilinear elliptic equations of the form

$$\begin{cases} -\Delta_p u = a(\delta(x))g(u) + f(x,u) + \lambda|\nabla u|^{p-1}, & \text{in } B_R \\ u > 0, & \text{in } B_R \\ u = 0, & \text{on } \partial B_R \end{cases} \tag{1.1}$$

where $B_R(0) \subset \mathbf{R}^N$, $N \geq 2$ is an open ball centered at origin of $\mathbf{R}^N$, g is an unbounded decreasing function, $a(\delta(x))$ is positive and continuous, $\delta(x) = \text{dist}(x, \partial B_R)$, $p \geq 2$, $\lambda < 0$. We emphasis the effect of all these terms in the study of existence and nonexistence of positive solutions.

Keywords: Gradient terms; Quasilinear elliptic equation; Singular; Existence and Nonexistence
2010 Mathematics Subject Classification: 35J15, 35J75.

1 Introduction

In this paper, we are concerned with quasilinear elliptic problems in the from

$$\begin{cases} -\triangle_p u = a(\delta(x))g(u) + f(x,u) + \lambda|\nabla u|^{p-1}, & \text{in } B_R \\ u > 0, & \text{in } B_R \\ u = 0, & \text{on } \partial B_R \end{cases} \tag{1.1}$$

where $B_R \subset \mathbf{R}^N, N \geq 2, \triangle_p u = \text{div}(|\nabla u|^{p-2}\nabla u)$, where B_R is a smooth and bounded domainis an open ball centered at origin of $\mathbf{R}^N$, $a((\delta(x))$ is positive and continuous, $p \geq 2$, $\lambda < 0$. For the convenience, we denote $h(x, u, \nabla u) = a(\delta(x))g(u) + f(x, u) + \lambda|\nabla u|^{p-1}$, and $h(x, u, \nabla u)$ is nonincreasing respect to u.

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We assume that $g \in C^1(0,\infty)$ is a positive decreasing function and

$(g1)$ $\lim_{t\to 0^+} g(t) = \infty$

The function $f : \bar{B}_R \times [0,\infty) \to [0,\infty)$ is $H\ddot{o}lder$ continuous, nondecreasing with respect to the second variable and f fulfills the hypotheses:

$(f1)$ the mapping $(0,\infty) \ni t \mapsto \frac{f(x,t)}{t^{p-1}}$ is nonincreasing for all $x \in \bar{B}_R$;

$(f2)$ $\lim_{t\to 0^+} \frac{f(x,t)}{t^{p-1}} = \infty$ and $\lim_{t\to\infty} \frac{f(x,t)}{t^{p-1}} = 0$, uniformly for $x \in \bar{B}_R$.

Such singular boundary value problems arise in the context of chemical heterogeneous catalysts and chemical catalyst, in the theory of heat conduction in electrically conducting materials, singular minimal surfaces, as well as in the study of non-Newtonian fluid theory [1], non-Newtonian filtration [2] and the turbulent flow of a gas in porous medium. In the non-Newtonian fluid theory, the quantity p is characteristic of the medium. Media with $p > 2$ are called dilatant fluids and those with $p < 2$ are called pseudoplastics. If $p = 2$, then they are Newtonian fluids.

The main features of the paper are the presence of the convection term $|\nabla u|^{p-1}$ combined with the singular weight $a : (0,\infty) \to (0,\infty)$ which is assumed to be nonincreasing and $H\ddot{o}lder$ continuous.

Many papers have been devoted to the case $a \equiv 1$ or $\lambda = 0$ ([3,4,5,6,7,8,9,10,11,12,13,14,15,16,17, 29] and the references therein). In [17], the author considered the following problem

$$\text{div}(|\nabla u|^{p-2}\nabla u) + q(x)u^{-\gamma} = 0, \ \ x \in \mathbf{R}^N, \tag{1.2}$$

has a positive entire solution if $1 < p < N,\ 0 \le \gamma < p-1$, and $q(x) \in C(\mathbf{R}_+)$ satisfy some suitable conditions.

In [3], the author studied the existence of the positive solutions of the equation

$$\begin{cases} -\triangle_p u = \lambda f(x,u), & \text{in } \Omega \\ u > 0, & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases} \tag{1.3}$$

where $\Omega \subset \mathbf{R}^N$ is a $C^{1,\omega}$ bounded domain, for some $0 < \omega < 1$, $f : \Omega \times (0,\infty) \to [0,\infty)$ is a suitable function and allowed to be singular, $\lambda > 0$.

In [5], the existence and uniquness of positive solutions of the following quasilinear singular equations

$$\begin{cases} -\triangle_p u = \rho(x)f(u), & \text{in } \mathbf{R}^N \\ u > 0, & \text{in } \mathbf{R}^N \\ \lim_{|x|\to\infty} u = 0. \end{cases} \tag{1.4}$$

and

$$\begin{cases} -\triangle_p u = \rho(x)f(u), & \text{in } \Omega \\ u > 0, & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases} \tag{1.5}$$

are considered.

One of the works in the literature dealing with singular weights in connection with singular nonlinearities is due to [11,16]. In [11,16], the following problem has been considered

$$\begin{cases} -(\varphi_p(u'(t)))' = a(t)g(u(t)), & \text{in } (0,1) \\ u > 0, & \text{in } (0,1) \\ u(0) = u(1) = 0 \end{cases} \tag{1.6}$$

where g satisfies $(g1)$ and a is positive and continuous on $(0,1)$. It is shown that if

$$\int_0^\delta \varphi_p^{-1}(\int_s^\delta a(\tau)d\tau)ds + \int_\delta^1 \varphi_p^{-1}(\int_\delta^s a(\tau)d\tau)ds < \infty, \tag{1.7}$$

where $0 < \delta < 1$, then (1.6) may be a positive one classical solution. In our framework, g is continuous at $t = 1$, so (1.7) reads as

$$\int_0^1 \varphi_p^{-1}(\int_s^1 a(\tau)d\tau)ds < \infty, \tag{1.8}$$

where $\varphi_p^{-1}(t) = |t|^{\frac{1}{p-1}}$.

When $p = 2$, there is a vast literature on stable solutions to the equation (1.1), we refer to [18,19,20,21,22,23,24] and references therein. In particular, Marius Ghergu and Taliaferro S.D. [20,22],[22] dealing with singular weights in connection with singular nonlinearities. The present paper is an part extension of [20,22] to the $p-$Laplacian equation. Our main ideas come from the paper [20,22].

Theorem 1.(Nonexistence) Suppose that $(g1)$, and

$(g2)$ $\forall\delta > 0, \Omega_\delta \supset \{x \in \Omega; \delta(x) < \delta\}$, such that

$$\int_{\Omega_\delta} G_p^1(a(\delta(x)))dx = \infty,$$

where G_p^1 is the inverse operator of $A_p^1 = -\triangle_p$.

Then (1.1) has no solutions.

Theorem 2. Assume $(f1)$, $(f2)$, and for some $R > 0$, then the problem (1.1) has at least one solution for all $\lambda < 0$.

2 Preliminary

Definition 2.1. A function $\underline{u} \in C^{1+\alpha}(\Omega) \cap C(\overline{\Omega})$ is called a subsolution of problem (2.3) if

$$-\text{div}(|\nabla\underline{u}|^{p-2}\nabla\underline{u}) \le h(x, \underline{u}, \nabla\underline{u}), \quad \underline{u} > 0, x \in \Omega, \underline{u} = 0, x \in \partial\Omega.$$

A function $\overline{u} \in C^{1+\alpha}(\Omega) \cap C(\overline{\Omega})$ is called a supersolution of problem(2.3)if

$$-\text{div}(|\nabla\overline{u}|^{p-2}\nabla\overline{u}) \ge h(x, \overline{u}, \nabla\overline{u}), \quad \overline{u} > 0, x \in \Omega, \overline{u} = 0, x \in \partial\Omega.$$

Lemma 2.1.([25],Theorem 9.5.])

$$Q(u, \phi) = \int_\Omega (A(x, u, \nabla u)) \cdot \nabla\phi - B(x, u, \nabla u)\phi)dx.$$

for all non-negative $\phi \in C_0^1(\Omega)$. Let u,v$\in C^1(\overline{\Omega})$ satisfy $Qu \ge 0$ in Ω, $Qv \le 0$ in Ω and $u \le v$ on $\partial\Omega$, where the functions A, B are continuously differentiable with respect to the z, p variables in $\overline{\Omega} \times \mathbf{R} \times \mathbf{R}^N$, the operator Q is elliptic in Ω, and the function B is non-increasing in z for fixed $(x, p) \in \Omega \times \mathbf{R}^N$. Then, if either

(i) the vector function A is independent of z;or

(ii) the function B is independent of p.

It follows that $u \le v$ in Ω.

Lemma 2.2.([26], Lemma 2.2]) Let $h(x, u, \xi)$ satisfy the following two basic conditions:

(A) $h(x, u, \xi)$ is locally Holder continuous function in $\Omega \times \mathbf{R}^+ \times \mathbf{R}^N$ and continuously differentiable with respect to the variables u and ξ;

(B) For every bounded domain $\Omega_1 \subset\subset \Omega$, for any $M > 0$, $\exists \rho(\Omega_1, M) > 0$, such that

$$|h(x, u, \xi)| < \rho(\Omega, M)(1 + |\xi|^p), x \in \Omega_1, 0 \leq u \leq M, \xi \in \mathbf{R}^N.$$

If problem (1.1) has a supersolution $\overline{u} \in C^1(\Omega)$ and a subsolution $\underline{u} \in C^1(\Omega)$ such that $\underline{u} \leq \overline{u}$ in Ω, the problem (1.1) has at least one solution $u(x) \in C^1(\Omega)$ with $\underline{u}(x) \leq u(x) \leq \overline{u}(x)$.

3 Proof of Theorem 1.

The proof of Theorem 1 follows from the following more general result.

Proposition 1. Assume that $(g1)$ and $(g2)$. Then the inequality boundary problem

$$\begin{cases} -\triangle_p u + \lambda(p-1)|\nabla u|^p \geq a(\delta(x))g(u), & \text{in } \Omega \\ u > 0, & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases} \tag{3.1}$$

(3.1) has no classical solutions.

Proof. Let (λ_1, φ_1)be the first eigenvalue and eigenfunction of $-\triangle_p$ in Ω subject to a homogenous Dirichlet boundary condition. It is known that $\lambda_1 > 0$ and by normalization, one can assume $\varphi_1 > 0$ in Ω. It suffices to prove the result only for $\lambda > 0$. We argue by contradiction and assume that there exists $u \in C^{1+\alpha}(\Omega) \cap C(\overline{\Omega})$ a solution of (3.1). Using $(g1)$, we can find $c_1 > 0$ such that $\underline{u} := c_1\varphi_1^{p-1}$ verifies

$$-\triangle_p u + \lambda(p-1)|\nabla u|^p \leq a(\delta(x))g(u), \text{ in } \Omega.$$

By comparison principle, we easily obtain

$$u \geq \underline{u}, \text{ in } \Omega \tag{3.2}$$

We make in (3.1) the change of variable $v = 1 - e^{-\lambda u}$. Therefore

$$\begin{cases} -\triangle_p v = \lambda^{(p-1)}(1-v)^{p-1}(\lambda(p-1)|\nabla u|^p - \triangle_p u) \geq \lambda^{(p-1)}(1-v)^{p-1}a(\delta(x))g(\frac{\ln(1-v)}{-\lambda}), & \text{in } \Omega \\ u > 0, & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega. \end{cases} \tag{3.3}$$

In order to avoid the singularities in (3.3), let us consider the approximated problem

$$\begin{cases} -\triangle_p v = \lambda^{(p-1)}(1-v)^{p-1}a(\delta(x))g(\epsilon - \frac{\ln(1-v)}{\lambda}), & \text{in } \Omega \\ u > 0, & \text{in } \Omega \\ u = 0, & \text{0n } \partial\Omega \end{cases} \tag{3.4}$$

with $0 < \epsilon < 1$, clearly v is a supersolution of (3.4). By (3.2) and the fact that $\lim_{t\to 0^+} \frac{1-e^{-\lambda t}}{t} = \lambda > 0$, there exist $c_2 > 0$ such that $v \geq c_2\varphi^{p-1}$ in Ω. On the other hand, there exist $0 < c < c_2$ such that $c\varphi^{p-1}$ is a subsolution of (3.4) and obviously $c\varphi^{p-1} \leq v$ in Ω. Then, by the standard sub- and sup-solution method, the problem (3.4) has a solution $v_\epsilon \in C^{1+\alpha}(\bar{\Omega})$ such that

$$c\varphi^{p-1} \leq v_\epsilon \leq v, \text{in } \Omega \tag{3.5}$$

From (3.4), we have

$$v_\epsilon = G_p^1(\lambda^{(p-1)}(1-v_\epsilon)^{p-1}a(\delta(x))g(\epsilon - \frac{\ln(1-v_\epsilon)}{\lambda}))$$

where G_p^1 is the inverse operator of $A_p^1 = -\triangle_p$ under the Dirichlet boundary condition.
So,

$$\int_\Omega v_\epsilon dx = \int_\Omega G_p^1(\lambda^{(p-1)}(1-v_\epsilon)^{p-1}a(\delta(x))g(-\frac{\ln(1-v_\epsilon)}{\lambda}))dx$$

Using (3.5), we obtain

$$\begin{array}{rcl} M & = & \int_\Omega v dx \\ & \geq & \int_\Omega G_p^1(\lambda^{(p-1)}(1-v)^{p-1}a(\delta(x))g(-\frac{\ln(1-v)}{\lambda}))dx \\ & \geq & C\int_{\Omega_\delta} G_p^1(a(\delta(x)))dx \end{array}$$

where $\Omega_\delta \supset \{x \in \Omega; \delta(x) < \delta\}$, for some $\delta > 0$ sufficient small. Since

$$\int_{\Omega_\delta} G_p^1(a(\delta(x)))dx = \infty,$$

by the above inequality, we find a contradiction. Hence, problem (3.1) has no classical solutions and the proof is now completed.

4 Proof of Theorem 2.

Let us note first that in our setting problem (1.1) reads

$$\begin{cases} -\triangle_p u = a(R-|x|)g(u) + f(x,u) + \lambda|\nabla u|^{p-1}, & |x| < R \\ u > 0, & |x| < R \\ u = 0, & |x| = R. \end{cases} \tag{4.1}$$

In order to provide a supersolution of (4.1), we consider the problem

$$\begin{cases} -\triangle_p u = a(R-|x|)g(u) + 1 + \lambda|\nabla u|^{p-1}, & |x| < R \\ u > 0, & |x| < R \\ u = 0, & |x| = R. \end{cases} \tag{4.2}$$

Lemma 4.1. Assume $(g1)$, problem (4.2) has at least one solution.

Proof. We are looking for radially decreasingly symmetric solution u of (4.2), that is $u = u(r)$, $0 \leq r = |x| \leq R$. In this case, problem (4.2) becomes

$$\begin{cases} -[(|u'|^{p-2}u')' + \frac{N-1}{r}|u'|^{p-2}u'] = a(R-|x|)g(u(r)) + 1 + \lambda|u'|^{p-1}, & |x| < R \\ u(r) > 0, & |x| < R \\ u(r) = 0, & |x| = R. \end{cases} \tag{4.3}$$

Since $u(r)$ is decreasing, that is $u'(r) \leq 0$, then (4.3) gives

$$-[(|u'|^{p-2}u')' + \frac{N-1}{r}|u'|^{p-2}u' - \lambda|u'|^{p-2}u'] = a(R-|x|)g(u(r)) + 1, \quad 0 \leq r < R.$$

We obtain

$$-(e^{-\lambda r}r^{N-1}|u'|^{p-2}u')' = e^{-\lambda r}r^{N-1}\psi(r,u(r)), \quad 0 \leq r < R, \tag{4.4}$$

where

$$\psi(r,t) = a(R-|x|)g(t) + 1, \quad (r,t) \in [0,R) \times (0,\infty).$$

From (4.4) we obtain

$$u(r) = u(0) - \int_0^r [e^{\lambda t} t^{1-N} \int_0^t e^{-\lambda s} s^{N-1} \psi(s, u(s)) ds]^{\frac{1}{p-1}} dt, \quad 0 \leq r < R. \tag{4.5}$$

On the other hand, due to [13] and to the symmetry of the domain, there exists a solution $\omega = \omega(r) \in C^{1+\alpha}(B_R(0)) \cap C(\bar{B_R}(0))$ of the problem

$$\begin{cases} -\triangle_p \omega = a(R - |x|)g(\omega) + 1, & |x| < R \\ \omega > 0, & |x| < R \\ \omega = 0, & |x| = R \end{cases} \tag{4.6}$$

As above we get

$$\omega(r) = \omega(0) - \int_0^r [t^{1-N} \int_0^t s^{N-1} \psi(s, \omega(s)) ds]^{\frac{1}{p-1}} dt, \ \ 0 \leq r < R. \tag{4.7}$$

We claim that there exists a solution $v \in C^{1+\alpha}(B_R(0)) \cap C(\bar{B_R}(0))$ of (4.5) such that $v > 0$ in $[0, R)$.

Let $A = \omega(0)$ and define the sequence $(v_k)_{k \geq 0}$ by

$$\begin{cases} v_k(r) = A - \int_0^r [e^{\lambda t} t^{1-N} \int_0^t e^{-\lambda s} s^{N-1} \psi(s, v_{k-1}(s)) ds]^{\frac{1}{p-1}} dt, & 0 \leq r < R \\ v_0(r) = \omega. \end{cases} \tag{4.8}$$

Since

$$\begin{aligned} v_1(r) &= A - \int_0^r [e^{\lambda t} t^{1-N} \int_0^t e^{-\lambda s} s^{N-1} \psi(s, v_0(s)) ds]^{\frac{1}{p-1}} dt \\ &\geq A - \int_0^r [e^{\lambda t} t^{1-N} e^{-\lambda t} \int_0^t s^{N-1} \psi(s, v_0(s)) ds]^{\frac{1}{p-1}} dt \\ &= v_0(r) \end{aligned}$$

we have $\omega = v_0(r) \leq v_1(r)$, then

$$\begin{aligned} v_2(r) &= A - \int_0^r [e^{\lambda t} t^{1-N} \int_0^t e^{-\lambda s} s^{N-1} \psi(s, v_1(s)) ds]^{\frac{1}{p-1}} dt \\ &\geq A - \int_0^r [e^{\lambda t} t^{1-N} \int_0^t e^{-\lambda s} s^{N-1} \psi(s, v_0(s)) ds]^{\frac{1}{p-1}} dt \\ &= v_1(r) \end{aligned}$$

As the above iteration, we reduce $v_k(r) \geq v_{k-1}(r)$ for all $k \geq 2$. Hence

$$\omega = v_0 \leq v_1 \leq v_2 \leq \cdots \leq v_k \leq \cdots \leq A, \ \text{ in } B_R(0).$$

Thus, there exists $\overline{v}(r) := \lim_{k \to \infty} v_k(r)$, for all $0 \leq r < R$ and $v > 0$ in $[0, R)$. We now can pass to the limit in (4.8) in order to get that v is a solution of (4.5) . By classical regularity results we also obtain $\overline{v} \in C^{1+\alpha}([0, R)) \cap C([0, R])$. This proves the claim.

Clearly, $\underline{v} = \overline{v} - \overline{v}(R)$ is a subsolution of (4.2). On the other hand, we have obtained a supersolution $\overline{v}$ of (4.2) such that $\overline{v} > \underline{v}$ in $B_R(0)$. So, by the standard sub- and sup-solution method, the problem (4.2) has at least one solution and the proof of Lemma 4.1 is completed.

Proof of Theorem 2. In order to provide a subsolution of (4.1), we consider the following problem

$$\begin{cases} -\triangle_p u = a(R - |x|)g(u) + \lambda |\nabla u|^{p-1}, & \text{in } |x| < R \\ u > 0, & \text{in } |x| < R \\ u = 0, & \text{on } |x| = R \end{cases} \tag{4.9}$$

It is easy to see that the solution of (4.9) is the subsolution of (4.1), next we are looking for the solution of (4.9).

In [6], it is easy to see that there exists $\overline{u}_1 \in C^{1+\alpha}(B_R) \cap C(\overline{B}_R)$ such that

$$\begin{cases} -\triangle_p u = a(R-|x|)g(u), & \text{in } |x| < R \\ u > 0, & \text{in } |x| < R \\ u = 0, & \text{on } |x| = R \end{cases} \tag{4.10}$$

It is obvious that $\overline{u}_1$ is a supsolution of (4.9) for all $\lambda < 0$.

We can obtain easily that there exists $\underline{u}_1$ such that

$$\begin{cases} -\triangle_p u = \lambda|\nabla u|^{p-1}, & \text{in } B_R(0) \\ u > 0, & \text{in } B_R(0) \\ u = 0, & \text{on } \partial B_R(0). \end{cases} \tag{4.11}$$

It is obvious that $\underline{u}_1 > 0$ is a subsolution of (4.9) for all $\lambda < 0$. By comparison principle, we easily obtain

$$\overline{u}_1 > \underline{u}_1.$$

Then, by the standard sub- and sup-solution method, the problem (4.9) has a solution u̲ and $\overline{u}_1 > \underline{u} > \underline{u}_1$. It follows that u̲ is a subsolution of (4.1).

Let u be a solution of (3.2). For $M > 1$ we have

$$\begin{aligned} -\triangle_p (Mu) = &\quad M^{p-1}a(R-|x|)g(u) + M^{p-1} + \lambda|\nabla(Mu)|^{p-1} \\ \geq &\quad a(R-|x|)g(Mu) + M^{p-1} + \lambda|\nabla(Mu)|^{p-1}. \end{aligned}$$

Since (f_1), we can choose $M > 1$ such that

$$M^{p-1} \geq f(x, M|u|_\infty), \text{in } B_R(0).$$

Then $\bar{u}_\lambda := Mu$ satisfies

$$-\triangle_p (\bar{u}_\lambda) \geq a(R-|x|)g(\bar{u}_\lambda) + f(x,\bar{u}_\lambda) + \lambda|\nabla(\bar{u}_\lambda)|^{p-1}, \text{ in } B_R(0).$$

It follows that $\bar{u}_\lambda$ is a supersolution of (4.1). Since Lemma 2.1, we know $\underline{u} \leq \bar{u}_\lambda$ in $B_R(0)$. So, (4.1) has at least one solution.

The proof of Theorem 1 is completed.

5 Conclusions

The boundary value quasilinear differential equation (1.1) are mathematical models occurring in the studies of the p-Laplace equation, generalized reaction-diffusion theory, non-Newtonian fluid theory, and the turbulent flow of a gas in porous medium. In the non-Newtonian fluid theory, the quantity p is characteristic of the medium. Media with $p > 2$ are call dilatant fluids and those with $p < 2$ are called pseudoplastics. If $p = 2$, they are Newtonain fluids. When $p \neq 2$, the problem becomes more complicated since certain nice properties in herent to the case $p = 2$ seem to be lost or at least difficult to verify. The main differences between $p = 2$ and $p \neq 2$ can be founded in [23,24]. When $p = 2$, it is well known that all the positive solutions in $C^2(B_R)$ of the problem

$$\begin{cases} \triangle u + f(u) = 0 \text{ in } B_R \\ u(x) = 0 \text{ on } \partial B_R \end{cases}$$

are radially symmetric solutions for very general f(see [27]). Unfortunately, this result does not apply to the case $p \neq 2$. Kichenassary and Smoller showed that there exist many positive nonradial solutions of the above problem for some f (see [28]). The major stumbling block in the case of $p \neq 2$ is that certain nice features inherent to the case $p = 2$ seem to be lost or at least difficult to verify. In this paper, we first give some necessary preliminary knowledge. Secondly, we further study the non-existence of positive solutions to problem (1.1) which the right hand side functions are singular with gradient terms based on the method of sub-supersolution. Finally, we consider the existence of positive solutions for singular quasilinear elliptic equations with gradient terms for (1.1) in B_R.

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Competing Interests

The authors declare that no competing interests exist.

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