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Estimation of Stress Strength Reliability in Single Component Models for Different Distributions

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Authors' contributions

This work was carried out in collaboration among all authors. Authors NB and TRJ designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors RB and SAM managed the analyses of the study and literature searches. All authors read and approved the final manuscript.

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ABSTRACT

This paper aims to estimate the stress-strength reliability parameter $R = P(Y < X)$, considering the two different cases of stress strength parameters, when the strength 'X' follows exponentiated inverse power Lindley distribution ,extended inverse Lindley and Stress 'Y' follows inverse power Lindley distribution and inverse Lindley distribution. The method of maximum likelihood estimation is used to obtain the reliability estimators. Illustrations are provided using R programming.

Keywords: Lindley distribution (LD); Inverse Lindley distribution (ILD); Inverse Power Lindley distribution (IPLD); Extended Inverse Lindley distribution (EILD); Exponentiated Inverse Power Lindley distribution (EIPLD); Maximum likelihood estimator (MLE).

1. INTRODUCTION

The LD was first introduced by D. V. Lindley [1]. The distribution is a mixture of the gamma distribution, with shape parameter 2 and scale parameter β and exponential distribution with scale parameter β . Its probability distribution function (pdf) is given by

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$$f(x;\beta)=\frac{\beta^2}{1+\beta^2}(1+x)e^{-\beta x}\;\; x>0,\beta>0.$$

The corresponding cumulative distribution function (cdf) is given by:

$$F(x;\beta)=1-\left(1+\frac{\beta}{1+\beta}x\right)e^{-\beta x}. \qquad x>0,\beta>0.$$

Since LD is only appropriate for modeling the data with monotonic increasing failure rate, its relevance may be restrained to the data that show non-monotonic shapes (bathtub and upside down bathtub) for their failure rates. Therefore, LD has been extended to various ageing classes and introduced various generalized class of lifetime distribution based on Lindley distribution. H. Zakerzadeh and A. Dolati [2] introduced three parameters extension of the Lindley distribution. S. Nadarajah et al. [3], M. E. Ghitany et al. [4] proposed two parameter generalizations of the LD, called as the generalized Lindley and power Lindley distributions. These distributions are generated using the exponentiation and power transformations to the Lindley distribution. F. Merovci [5] investigated transmuted Lindley and transmuted Lindley-geometric distributions respectively. The exponentiated power Lindley distribution (EPLD) was introduced by S. K. Ashour and M. A. Eltehiwy [6].

In the above cited reference, the authors mainly fixate on the estimation of increasing, decreasing and bathtub shaped failure rate. V. K. Sharma, S. K. Singh and U. Singh [7] proposed a lifetime model with upside-down bathtub shape hazard rate function that is efficient of modeling many real problems, for example failure of washing machines, survival of head and neck cancer patients, and survival of patients with breast cancer. Considering the fact that all inverse distribution acquire the upside-down bathtub shape for their hazard rates, V. K. Sharma, S. K. Singh, U. Singh and V. Agiwal [8], proffered an inverted version of the LD that can be used to model the upside-down bathtub shape hazard rate data.

The ILD take into account the inverse of a random variable with a LD. If a random variable Y has a LD, then a random variable Y=1/X follows ILD with probability distribution function defined by

$$f(y;\beta)=\frac{\beta^2}{1+\beta^2}\left(\frac{1+x}{x^3}\right)e^{-\frac{\beta}{x}} \qquad ;x>0,\beta>0.$$

The corresponding cumulative distribution function (cdf) is given as:

$$F(y;\beta)=\left(1+\frac{\beta}{1+\beta}\frac{1}{x}\right)e^{-\frac{\beta}{x}}. \qquad ;x>0,\beta>0.$$

In order to accomplish more flexible family of distributions, another generalization is the IPLD suggested by Barco, Mazucheli and Janeiro [9] by considering the power transformation, $X=Y^{\frac{1}{\alpha}}$. Explicitly if a random variable Y follows ILD, then the random variable $Z=Y^{\frac{1}{\alpha}}$ follows IPLD with density and cumulative distribution functions defined respectively as

$$f(z;\alpha,\beta)=\frac{\alpha\beta^2}{1+\beta}\left(\frac{1+z^\alpha}{z^{2\alpha+1}}\right)e^{-\frac{\beta}{z^\alpha}}\; ;x>0,\alpha>0,\beta>0.$$

$$F(z;\alpha,\beta)=\left(1+\frac{\beta}{1+\beta}\frac{1}{z^\alpha}\right)e^{-\frac{\beta}{z^\alpha}}. \qquad ;x>0,\alpha>0,\beta>0.$$

A new extension of ILD was given by V.K.Sharma and Khandelwal [10], known as EILD which deals with more malleability with the effective shape parameter. Its probability density function (pdf) is given by:

$$f(x;\alpha,\beta)=\frac{\beta^\alpha}{(1+\beta)\Gamma\alpha}\frac{(1+x(\alpha-1))}{x^{\alpha+1}}e^{-\frac{\beta}{x}}$$
$$;x>0,\beta>0,\alpha>1.$$

The analogous cumulative distribution function (cdf) is given by:

$$F(x;\alpha,\theta)=pCDFInverseGamma(\alpha-1,\beta)+(1-p)CDFInverseGamma(\alpha,\beta) \qquad \alpha>1\;;\beta>0.$$

Where, $CDFInverseGamma(\alpha,\beta)=\frac{\Gamma(\beta/x,\alpha)}{\Gamma\alpha}$, $\Gamma(t,\alpha)=\int_t^\infty x^{\alpha-1}e^{-x}dx$ and $\Gamma\alpha=\int_0^\infty t^{(s-1)}e^{-t}dt$.

A new three parameter probability distribution introduced by R. Jan et al. [11] known as Exponentiated inverse power Lindley distributed (EIPLD). Its pdf and cdf is given by:

$$f(x)=\frac{\alpha\beta^2\gamma}{1+\beta}\left(\frac{1+x^\alpha}{x^{2\alpha+1}}\right)e^{-\frac{\beta\gamma}{x^\alpha}}\left(1+\frac{\beta}{1+\beta}\frac{1}{x^\alpha}\right)^{\gamma-1}\qquad x>0,\quad \alpha,\beta,\gamma>0$$

$$F(x)=\left[\left(1+\frac{\beta}{1+\beta}\frac{1}{x^\alpha}\right)e^{-\frac{\beta}{x^\alpha}}\right]\qquad x>0,\quad \alpha,\beta,\gamma>0$$

The stress strength parameter plays an important role in the reliability analysis (Fig. 2). For example if X is the strength of a system which is subjected to stress Y, then the parameter *R* = *P* $(Y<X)$ measures the system performance and it is very common in the context of mechanical reliability of a system. Moreover, *R* provides the probability of a system failure, if the system fails whenever the applied stress is greater than its strength. Many authors developed the estimation procedures for estimating the stress–strength reliability from various lifetime models, see [12,13] and references cited therein. Recently, M. M. Mohie El-Din, A. Sadek and Sh. H. Elmeghawry [14] obtained Stress-Strength Reliability Estimation for Exponentiated Generalized Inverse Weibull Distribution. T. R. Rasethuntsa and M. Nadar [15] derived Stress–strength reliability of a non-identical-component-strengths system based on upper record values from the family of Kumaraswamy generalized distributions. A. Iranmanesh, K. F. Vajargah and M. Hasanzadeh [16] studied estimation of stress strength reliability parameter of inverted gamma distribution.

In this paper, we have addressed the problem of estimating *R* = *P*(*Y* < *X*) considering the two different cases for stress strength reliability

1) When stress follows IPLD and strength follows EIPLD.
2) When stress follows ILD and strength follows EILD.

2. RELIABILITY AND ITS MAXIMUM LIKELIHOOD FUNCTION

CASE 1: Let Y ~ IPLD(α,β_1) and X ~ EIPLD (α,β_2,γ) be independent random variables,

Suppose that X represent the strength of a component exposed to Y stress, then the stress strength reliability(SSR) of this component is obtained as follows,

$$\begin{aligned}
R=P(X>Y) &= \int_0^\infty p(X>Y/Y=y)f_y(y)dy\\
&= \int_0^\infty S_x(y)f_y(y)dy\\
&= \int_0^\infty\left(1+\frac{\beta_1}{1+\beta_1}\frac{1}{x^\alpha}e^{\frac{-\beta_1}{x^\alpha}}\right)^\gamma\frac{\alpha\beta_2^2}{1+\beta_2}\left(\frac{1+x^\alpha}{x^{2\alpha+1}}\right)e^{-\frac{\beta_2}{x^\alpha}}dx\\
&= \frac{\alpha\beta_2^2}{1+\beta_2}\int_0^\infty\left(1+\frac{\beta_1}{1+\beta_1}\frac{1}{x^\alpha}\right)^\gamma\left(\frac{1+x^\alpha}{x^{2\alpha+1}}\right)e^{-\frac{\beta_1\gamma+\beta_2}{x^\alpha}}dx\\
&= \frac{\alpha\beta_2^2}{1+\beta_2}\int_0^\infty\sum_{i=1}^\infty {}^\gamma C_i\left(\frac{\beta_1}{1+\beta_1}\right)^i\frac{1}{x^{i\alpha}}\left(\frac{1+x^\alpha}{x^{2\alpha+1}}\right)e^{-\frac{\beta_1\gamma+\beta_2}{x^\alpha}}dx
\end{aligned}$$

$$= \frac{\beta_2^2}{1+\beta_2}\sum_{i=1}^{\infty} {}^{\gamma}C_i \left(\frac{\beta_1}{1+\beta_1}\right)^i \left[\int_0^{\infty} t^{i+1} e^{-(\beta\gamma_1+\beta_2)} dt + \int_0^{\infty} t^i e^{-t(\beta_1\gamma+\beta_2)} dt\right]$$

$$R = \frac{\beta_2^2}{1+\beta_2}\sum_{i=1}^{\infty} {}^{\gamma}C_i \left(\frac{\beta_1}{1+\beta_1}\right)^i \frac{\Gamma(i+1)}{(\beta_1\gamma+\beta_2)^{i+1}}\left[\frac{i+1}{\beta_1\gamma+\beta_2}+1\right] \quad (1.1)$$

where R is independent of α

Suppose $x_1, x_2, ..., x_n$ is a random sample of size n_1 from EIPLD $(\alpha, \beta_1, \gamma)$ and $y_1, y_2, ... y_n$ is an independent random sample of size n_2 from IPL (α, β_2). The likelihood function l=l (θ) where $\theta = (\alpha, \beta_1, \beta_2, \gamma)$ based on the two independent random sample is given by:

$$l = \sum_{i=1}^{n_1} \ln\left[f_x(x_i)\right] + \sum_{j=1}^{n_2} \ln\left[f_y(y_j)\right]$$

$$l = n_1\left[\ln\alpha + 2\ln\beta_1 + \ln\gamma - \ln(1+\beta_1)\right] + \sum_{i=1}^{n_1}\ln(1+x_i{}^{\alpha}) - (2\alpha+1)\sum_{i=1}^{n_1}\ln x_i - \beta_1\gamma\sum_{i=1}^{n_1}\left(\frac{1}{x_i^{\alpha}}\right) + (\gamma-1)\sum_{i=1}^{n_1}\ln\left(1+\frac{\beta_1}{1+\beta_1}\frac{1}{x_i{}^{\alpha}}\right)$$

$$+ n_2\left[2\ln\beta_2 - \ln(1+\beta_2)\right] + \sum_{j=1}^{n_2}\ln(1+y_j) - 3\sum_{j=1}^{n_2}\ln(y_j) - \beta_2\sum_{j=1}^{n_2}\left(\frac{1}{y_j}\right)$$

The MLE $\hat{\theta}$ of θ is the solution of non-linear equations (1.2), (1.3), (1.4) (1.5)

$$\frac{\partial}{\partial\alpha} = \frac{n_1}{\alpha} + \sum_{i=1}^{n_1}\frac{x_i^{\alpha}\ln x_i}{1+x_i^{\alpha}} - 2\sum_{i=1}^{n_1}\ln x_i + \beta_1\gamma\sum_{i=1}^{n_1}x_i{}^{\alpha}\ln x_i + (\gamma-1)(1+\beta_1)\sum_{i=1}^{n_1}\frac{\beta_1}{1+2\beta_1}\ln\left(\frac{1}{x_i^{\alpha}}\right) = 0 \quad (1.2)$$

$$\frac{\partial l}{\partial\beta_1} = \frac{2n_1 + n_1\beta_1}{\beta_1(1+\beta_1)} - \gamma\sum_{i=1}^{n_1}\frac{1}{x^{\alpha}} + (\gamma-1)\sum_{i=1}^{n_1}\frac{x_i^{\alpha}}{(1+\beta_1)x_i^{\alpha}+\beta_1}\left(\frac{1}{x_i^{\alpha}}\frac{1}{(1+\beta_1)}\right) = 0 \quad (1.3)$$

$$\frac{\partial l}{\partial\beta_2} = \frac{2n_2}{\beta_2} - \frac{n_2}{1+\beta_2} - \sum_{j=1}^{n_2}\left(\frac{1}{y_j}\right) \quad (1.4)$$

$$= \frac{2n_2 + 2n_2\beta_2 - n_2\beta_2}{\beta_2(1+\beta_2)} - \sum_{j=1}^{n_2}\frac{1}{y_j} = 0$$

$$= -\beta_2{}^2\sum_{i=1}^{n_2}\frac{1}{y_j} - \beta_2\left[\sum_{j=1}^{n_2}\frac{1}{y_j} - n_2\right] + 2n_2 = 0$$

$$\hat{\beta}_2 = \beta_2(\hat{\alpha}) = -\frac{\left(\sum_{j=1}^{n_2}\frac{1}{y_j} - n_2\right) + \sqrt{\left(\sum_{j=1}^{n_2}\frac{1}{y_j} - n_2\right)^2 + 8n_2\sum_{j=1}^{n_2}\frac{1}{y_j}}}{2\sum_{j=1}^{n_2}\left(\frac{1}{y_j}\right)}$$

$$\frac{\partial l}{\partial \gamma} = \frac{n_1}{\gamma} - \beta_1 \sum_{i=1}^{n_1} \frac{1}{x_i^{\alpha}} + \sum_{i=1}^{n_1} \ln\left(1 + \frac{\beta_1}{1+\beta_1} \frac{1}{x_i^{\alpha}}\right) = 0 \tag{1.5}$$

$$\hat{\gamma} = \gamma(\hat{\alpha}) = \frac{n_1}{\beta_1 \sum_{i=1}^{n_1} \frac{1}{x_i^{\alpha}} + \sum_{i=1}^{n_1} \ln\left(1 + \frac{\beta_1}{1+\beta_1} \frac{1}{x_i^{\alpha}}\right)}$$

Where $\hat{\alpha}$ is the solution of non linear equation :

$$G(\alpha) = \frac{n_1}{\alpha} + \sum_{i=1}^{n_1} \frac{x_i^{\alpha} \ln x_i}{1 + x_i^{\alpha}} - 2\sum_{i=1}^{n_1} \ln x_i + \beta_1(\hat{\alpha}) \sum_{i=1}^{n_1} x_i^{\alpha} \ln x_i + (\gamma(\hat{\alpha}) - 1)(1 + \beta_1(\hat{\alpha})) \sum_{i=1}^{n_1} \frac{\beta_1(\hat{\alpha})}{1 + 2\beta_1(\hat{\alpha})} \ln\left(\frac{1}{x_i^{\alpha}}\right)$$

Case 2: Let X ~ EILD(α, β_1) and Y ~ ILD(β_2) be independent random variables, Suppose that X represent the strength of a component exposed to Y stress, then the Stress Strength Reliability (SSR) of this component is obtained as follows,

R= P(X>Y) = $\int_0^{\infty} p(X > Y / Y = y) f_y(y) dy$

$$R = \int_0^{\infty} \left(\int_0^{x} g(y) dy \right) f(x) dx$$

$$= \int_0^{\infty} \left(\int_0^{x} \frac{\beta_2^{\,2}}{1+\beta_2} \left(\frac{1+y}{y^3} \right) e^{-\frac{\beta_2}{y}} dy \right) \frac{\beta_1^{\alpha} e^{-\frac{\beta_1}{x}} [x(\alpha - 1) + 1]}{1 + \beta_1 x^{\alpha+1} \Gamma\alpha} dx$$

$$= \frac{\beta_1^{\alpha}}{1+\beta_1} \frac{1}{\Gamma\alpha} \int_0^{\infty} \left(1 + \frac{\beta_2}{1+\beta_2} \frac{1}{x} \right) \frac{e^{-\frac{1}{x}(\beta_1 + \beta_2)}}{x^{\alpha+1}} [x(\alpha - 1) + 1] dx$$

$$= \frac{\beta_1^{\alpha}}{1+\beta_1} \frac{1}{\Gamma\alpha} \left[(\alpha - 1) \int_0^{\infty} \frac{e^{-(\beta_1 + \beta_2)\frac{1}{x}}}{x^{\alpha}} dx + \int_0^{\infty} \frac{e^{-(\beta_1 + \beta_2)\frac{1}{x}}}{x^{\alpha+1}} dx + (\alpha - 1) \frac{\beta_2}{1+\beta_2} \int_0^{\infty} \frac{e^{-(\beta_1 + \beta_2)\frac{1}{x}}}{x^{\alpha+1}} dx + \frac{\beta_2}{1+\beta_2} \int_0^{\infty} \frac{e^{-(\beta_1 + \beta_2)\frac{1}{x}}}{x^{\alpha+2}} dx \right]$$

$$= \frac{\beta_1^{\alpha}}{1+\beta_1} \frac{1}{\Gamma\alpha} \left[\frac{(\alpha - 1)\Gamma(\alpha - 1)}{(\beta_1 + \beta_2)^{\alpha-1}} + \frac{\Gamma\alpha}{(\beta_1 + \beta_2)^{\alpha}} + \frac{(\alpha - 1)\beta_2}{1+\beta_2} \left(\frac{\Gamma\alpha}{(\beta_1 + \beta_2)^{\alpha}} \right) + \frac{\beta_2}{(1+\beta_2)} \frac{\Gamma(\alpha + 1)}{(\beta_1 + \beta_2)^{\alpha+1}} \right]$$

$$R = \frac{\beta_1^{\alpha}}{(1+\beta_1)(\beta_1 + \beta_2)^{\alpha+1}} \left[\frac{(1+\beta_2)(\beta_1 + \beta_2)^2 + (1+\beta_2)(\beta_1 + \beta_2) + (\alpha\beta_2 - \beta_2)(\beta_1 + \beta_2) + \alpha\beta_2}{(1+\beta_2)} \right] \tag{2.1}$$

Suppose $x_1, x_2, ..., x_n$ is a random sample of size n_1 from EILD (α, β_1) and $y_1, y_2, ... y_n$ is an independent random sample of size n_2 from ILD (β_2). The likelihood function $l = l\ (\theta)$ where $\theta = (\alpha, \beta_1, \beta_2)$ based on the two independent random sample is given by:

$$l = \sum_{i=1}^{n_1} \ln[f_x(x_i)] + \sum_{j=1}^{n_2} \ln[f_y(y_j)]$$

The MLE $\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2$ of α, β_1, β_2 is the solution of non-linear equations (2.2), (2.3), (2.4)

$$\frac{\partial}{\partial \alpha} l = n_1 \log \beta_1 + \sum_{i=1}^{n_1} \frac{x_i}{\left(x_i(\alpha-1)+1\right)} - \sum_{i=1}^{n_1} \log x_i - \frac{n}{\Gamma\alpha} \frac{\partial}{\partial x} \Gamma\alpha = 0 \tag{2.2}$$

$$\frac{\partial}{\partial \beta_1} l = n_1 \left(\frac{\alpha}{\beta_1} - \frac{1}{1+\beta_1} \right) - \sum_{i=1}^{n_1} \frac{1}{x_i} = 0 \tag{2.3}$$

$$\frac{\partial l}{\partial \beta_1} = \frac{n_1 \alpha}{\beta_1} - \frac{n_1}{1+\beta_1} - \sum_{i=1}^{n_1} \left(\frac{1}{x_i} \right) = 0$$

$$= \frac{n_1\alpha + n_1\alpha\beta_1 - n_1\beta_1}{\beta_1(1+\beta_1)} - \sum_{i=1}^{n_1} \frac{1}{x_i} = 0$$

$$= -\beta_1^{\,2} \sum_{i=1}^{n_1} \frac{1}{x_i} - \beta_1 \left[\sum_{i=1}^{n_1} \frac{1}{x_i} - n_1\alpha + n_1 \right] + n_1\alpha = 0$$

$$\hat{\beta}_1 = \beta_1(\hat{\alpha}) = -\frac{\left(\sum_{i=1}^{n_1} \frac{1}{x_i} - n_1\alpha + n_1 \right) + \sqrt{\left(\sum_{i=1}^{n_1} \frac{1}{x_i} - n_1\alpha \right)^2 + 4n_1\alpha \sum_{i=1}^{n_1} \frac{1}{x_i}}}{2\sum_{i=1}^{n_1} \left(\frac{1}{x_i} \right)}$$

$$\frac{\partial l}{\partial \beta_2} = \frac{2n_2}{\beta_2} - \frac{n_2}{1+\beta_2} - \sum_{j=1}^{n_2} \left(\frac{1}{y_j} \right) = 0 \tag{2.4}$$

$$= -\beta_2^{\,2} \sum_{i=1}^{n_2} \frac{1}{y_j} - \beta_2 \left[\sum_{j=1}^{n_2} \frac{1}{y_j} - n_2 \right] + 2n_2 = 0$$

$$\hat{\beta}_2 = \beta_2(\hat{\alpha}) = -\frac{\left(\sum_{j=1}^{n_2} \frac{1}{y_j} - n_2 \right) + \sqrt{\left(\sum_{j=1}^{n_2} \frac{1}{y_j} - n_2 \right)^2 + 8n_2 \sum_{j=1}^{n_2} \frac{1}{y_j}}}{2\sum_{j=1}^{n_2} \left(\frac{1}{y_j} \right)}$$

Where $\hat{\alpha}$ is the solution of non linear equation

$$G(\alpha) = n_1 \log \beta_1(\hat{\alpha}) + \sum_{i=1}^{n_1} \frac{x_i}{\left(x_i(\alpha-1)+1\right)} - \sum_{i=1}^{n_1} \log x_i - \frac{n}{\Gamma\alpha} \frac{\partial}{\partial x} \Gamma\alpha$$

Table 1. Stress strength reliability when stress follows IPLD and strength follows EIPLD

				$\gamma = 1$				
				β_1				
β_2		0.1	1.5	1.9	2.6	2.8	3.5	4.2
	0.2	0.2581	0.0151	0.0097	0.0053	0.0045	0.0029	0.0020
	1.3	0.0828	0.0964	0.0764	0.0528	0.0479	0.0351	0.0268
	2.5	0.0425	0.1004	0.0879	0.0690	0.0645	0.0514	0.0417
	4.6	0.0222	0.0809	0.0766	0.0673	0.0646	0.0559	0.0484
				$\gamma = 2$				
				β_1				
β_2		0.1	1.5	1.9	2.6	2.8	3.5	4.2
	0.2	0.3004	0.0078	0.0049	0.0026	0.0022	0.0014	0.0099
	1.3	0.1493	0.0812	0.0591	0.0369	0.0328	0.0227	0.0166
	2.5	0.0807	0.1085	0.0867	0.0605	0.0550	0.0406	0.0310
	4.6	0.0432	0.1078	0.0943	0.0739	0.0690	0.0549	0.0444
				$\gamma = 3$				
				β_1				
β_2		0.1	1.5	1.9	2.6	2.8	3.5	4.2
	0.2	0.2960	0.0052	0.0032	0.0017	0.0015	0.0009	0.0006
	1.3	0.2035	0.0666	0.0466	0.0279	0.0245	0.0166	0.0119
	2.5	0.1152	0.1019	0.0773	0.0507	0.0455	0.0323	0.0241
	4.6	0.0631	0.1156	0.0959	0.0700	0.0643	0.0488	0.0381

Table 2. Stress strength reliability when stress follows ILD and strength follows EILD

				$\gamma = 1$				
				β_1				
β_2		0.1	1.5	1.9	2.6	2.8	3.5	4.2
	0.2	0.2581	0.0151	0.0097	0.0053	0.0045	0.0029	0.0020
	1.3	0.0828	0.0964	0.0764	0.0528	0.0479	0.0351	0.0268
	2.5	0.0425	0.1004	0.0879	0.0690	0.0645	0.0514	0.0417
	4.6	0.0222	0.0809	0.0766	0.0673	0.0646	0.0559	0.0484
				$\gamma = 2$				
				β_1				
β_2		0.1	1.5	1.9	2.6	2.8	3.5	4.2
	0.2	0.3004	0.0078	0.0049	0.0026	0.0022	0.0014	0.0099
	1.3	0.1493	0.0812	0.0591	0.0369	0.0328	0.0227	0.0166
	2.5	0.0807	0.1085	0.0867	0.0605	0.0550	0.0406	0.0310
	4.6	0.0432	0.1078	0.0943	0.0739	0.0690	0.0549	0.0444
				$\gamma = 3$				
				β_1				
β_2		0.1	1.5	1.9	2.6	2.8	3.5	4.2
	0.2	0.2960	0.0052	0.0032	0.0017	0.0015	0.0009	0.0006
	1.3	0.2035	0.0666	0.0466	0.0279	0.0245	0.0166	0.0119
	2.5	0.1152	0.1019	0.0773	0.0507	0.0455	0.0323	0.0241
	4.6	0.0631	0.1156	0.0959	0.0700	0.0643	0.0488	0.0381

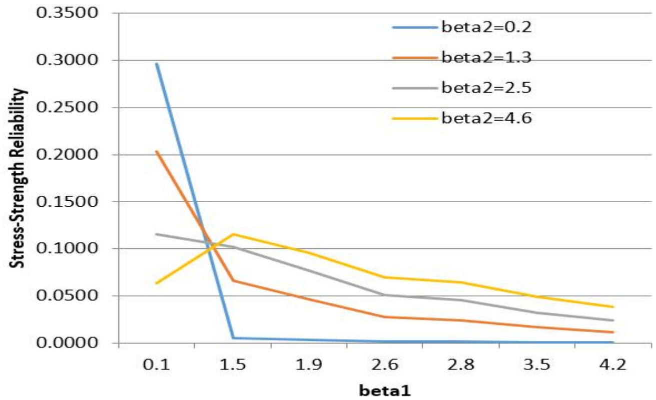

Fig. 1.1. Graphical overview of stress-strength reliability expression (1.1) for gamma=1 and different values of beta 1 and beta 2

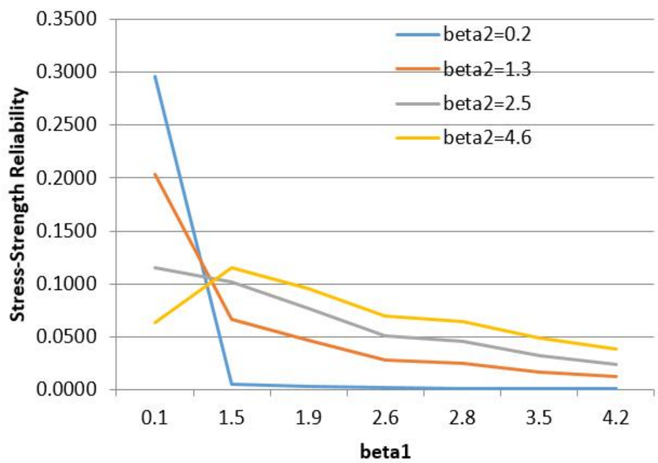

Fig. 1.2. Graphical overview of stress-strength reliability expression (1.1) for gamma=2 and different values of beta 1 and beta 2

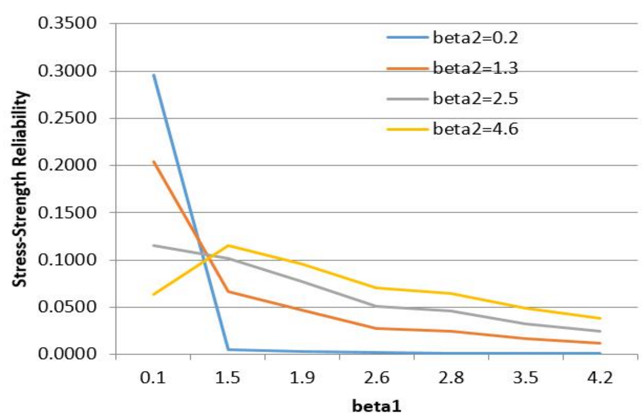

Fig. 1.3. Graphical overview of stress-strength reliability expression (1.1) for gamma=3 and different values of beta 1 and beta 2

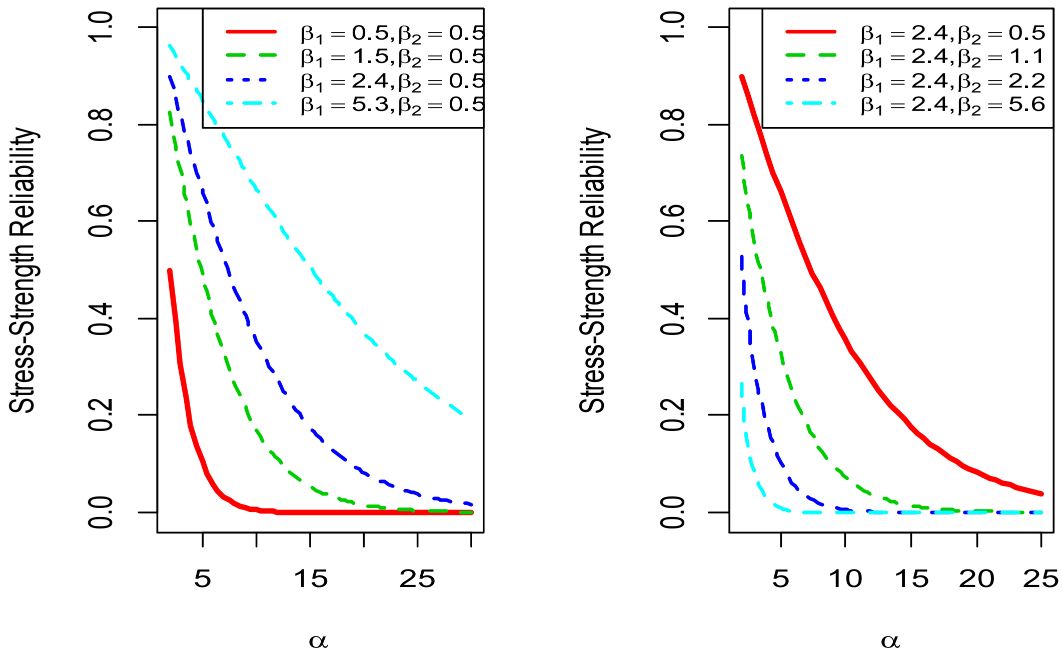

Fig. 2. Graphical overveiw of stress strength reliabilty for different values of parameters

3. CONCLUSION

In this paper, we have studied the stress strength reliability considering the two different cases of stress strength parameters. When strength X$\sim$ EIPLD (α, β_2, γ) and stress Y$\sim$IPL (α, β_1), it was observed that with increase in the value of strength parameter γ with fixed parameters (β_1, β_2), the stress strength reliability increases. However it is seen that, with increase in the value of stress parameter β_2, the stress strength reliability decreases keeping (γ, β_1) fixed (Table 1). Further the graphical overview of Stress strength reliability for $\gamma_1, \gamma_2, \gamma_3$ for different values of (β_1, β_2) are shown in Figs. (1.1, 1.2, 1.3) respectively. Also, when the strength X$\sim$ EILD(α, β_1) and Stress Y$\sim$ILD(β_2), it was found that as the value of stress parameter β_2 increases, keeping the strength parameters fixed (α, β_1), the stress strength parameter decreases. While as, with increase in the value of strength parameter β_1, the stress strength reliability increases, keeping α, β_2 fixed (Table 2). Hence we conclude that with decrease in the value of stress parameter and increase in value of strength parameter, reliability of single component system increases resulting in efficiency of system model.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Lindley DV. Fiducial distributions and Bayes theorem. Journal of the Royal Statistical Society. 1958;20:102-107.
2. Zakerzadeh H, Dolati A. A generalised Lindley distribution. Journal of Math Extension. 2009;3:13–25.
3. Nadarajah S, Bakouch H, Tahmasbi R. A generalized Lindley distribution. Sankhya B - Applied and Interdisciplinary Statistics. 2011;73:331–359.
4. Ghitany ME, Al-Mutairi DK, Aboukhamseen SM. Estimation of the reliability of a stress–strength system from power Lindley distributions. Communications in Statistics – Simulation and Computation. 2013;44(1):118-136.
5. Merovci F. Transmuted Lindley distribution. Int. J. Open Problems Compt. Math. 2013;6:63–7.
6. Ashour SK, Eltehiwy MA. Exponentiated power Lindley distribution. Journal of Advanced Research. 2015;6:895–905.

7. Sharma VK, Singh SK, Singh U. A new upside-down bathtub shaped hazard rate model for survival data analysis. Applied Mathematics and Computation. 2014;239: 242-253.
8. Sharma VK, Singh SK, Singh U, Agiwal V. The inverse lindley distribution: A stress strength reliability model. Journal of Industrial and Production Engineering. 2015;32(3):162-173.
9. Kelly Vanessa Parede Barco, Josmar Mazucheli, Vanderly Janeiro. The inverse power Lindley distribution. Communications in Statistics - Simulation and Computation. 2016;46(8):6308-6323.
10. Sharma VK, Pragya Khandelwal. On the extension of inverse Lindley distribution. Journal of Data Science. 2017;15:205-220.
11. Jan R, Bashir N, Bilal P, Jan TR. Exponentiated inverse power Lindley distribution and its application. Pakistan Journal of Statistics and Operations Research; 2018. (To Appear)
12. Sharma VK, Singh SK, Singh U, Agiwal V. The inverse Lindley distribution: A stress reliability model with application to head and neck cancer data. Journal of Industrial and Production Engineering. 2015;32:162–173.
13. Srinivasa GR. Estimation of reliability in multicomponent stress–strength model based on generalized exponential distribution. Colombian Journal of Statistics. 2012;35:67–76.
14. Mohie El-Din MM, Sadek A, Sh. H. Elmeghawry. Stress-strength reliability estimation for exponentiated generalized inverse weibull distribution. Journal of Statistics Applications & Probability. 2018;7(1):127-136.
15. Rasethuntsa TR, Nadar M. Stress–strength reliability of a non-identical-component-strengths system based on upper record values from the family of Kumaraswamy generalized distributions. A Journal of Theoretical and Applied Statistics. 2018;52(3):684-716.
16. Iranmanesh A, Vajargah KF, Hasanzadeh M. Estimation of stress strength reliability parameter of inverted gamma distribution. Mathematical Sciences. 2018;12:71–77.

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